Incentive Design for "Free" but "No Free Disposal" Services: The Case of Personalization under Privacy Concerns

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Abstract

Online personalization services belong to a class of economic goods with "no-free-disposal" (NFD) property, where due to privacy concerns more services is not always preferred to less. Therefore, even if these services are offered for free, online vendors find only a subset of services being used and thus acquire a reduced amount of preference and usage information. Vendors have invested in sophisticated technologies such as browser-embedded toolbars to have greater control in designing mechanisms for consumers' usage of these services. This paper analyzes a monopolist's optimal contract designs for personalization services under information asymmetry in a market where consumers are heterogeneous in their concern for privacy. Our analyses reveal several interesting results some of which are counter to those observed in other information goods market with zero marginal cost. Not surprisingly, under full information the vendor strictly prefers a variable to fixed contract as he can discriminate in the former and finds that under certain condition providing coupons or incentives with usage to be profit improving. However when considering services-only contracts under asymmetry, a fixed contract is optimal under certain conditions and surprisingly for some markets, a consumer-welfare maximizing contract emerges to be superior for the monopolist. An interesting aspect of the fixed contract is that when the vendor engages in couponing he continues to serve the same proportion of consumers as in the services-only case, even if consumer welfare, producer surplus and social welfare are all higher with incentives. We show that a purely usage-based contract is sub-optimal even when marginal costs are zero, as the monotonicity constraints on incentives become binding. As a result we develop a bunched solution that is weakly superior to all other contracts; interestingly our results shows that an optimal truth revealing contract requires more incentives need to be paid to consumers that value personalization over privacy than the intuitive view that couponing is effective as a tool to bring in privacy-seekers to participate in the market.

Keywords: Incentives, mechanism design, personalization, privacy, no free disposal

1 Introduction

Recent advances in Internet technologies have allowed online vendors to provide personalization through browser-embedded toolbars which allow them to automatically monitor usage information and provide various personalized offerings. A toolbar is an example of a Browser Helper Object (BHO), wherein once downloaded and embedded it has the ability monitor and report usage information (including Web sites visited, information filled in online forms, etc.) as well as tailor future Web browsing based on this behavior. Toolbars and personalization services accompanying them are generally available free of charge since the vendor's profit motive is based on the consumers' preference information acquired during the usage of these services. On the flip side, consumers' decision to avail personalization services is a rather tricky one in that while they are free of monetary cost and provide convenience, privacy concerns abound as preference and usage information is shared with a vendor. Thus while online vendors have been focused on getting consumers to share information through personalization offerings, the question, "does personalization jeopardize our privacy?" has been of great interest to the legal community and public at large (Volokh 2000). Consistent with the observation of Murthi and Sarkar (2003), who call for research in mechanism design for market where personalization and privacy co-exist, this paper examines various personalization strategies and incentive designs of online vendors who face consumers that are concerned about their privacy.

Personalization services belong to an interesting class of economic goods and services that are known to possess "no free disposal" (NFD) property. While somewhat less studied than their "free disposal" counterpart, we commonly come across these goods when users have an intrinsic disutility from consuming beyond their satiation level (Nahata, et al. 2003); the general principle being that more is not always preferred to less. Classic examples include a traveler's utility for package size of toiletries (larger sizes are not always optimal), consumers' preference for sweetness of a drink (over-sweetness may not be preferred) and a township's value for power production (production creates pollution) (Rothwell and Rust 1997). Given that consumers' satiation of personalization services is a function for their privacy costs (Chellappa and Sin 2005), deciding a vendor's optimal service offerings when consumers vary in this satiation level can be quite complex. This complexity is compounded due to the inability of personalization vendors to charge prices, and by the fact that, consumers' privacy costs are often private leading to market asymmetries for the vendor. Thus in responding to Rust, et al.'s (2002) observation that with increased technological sophistication a market for privacy is emerging, any mechanism design for such markets needs to explore some non-price instruments. Indeed some researchers have noted the importance of providing incentives to get consumers to share information (Resnick and Varian 1997), and thus raising the potential for couponing/rewards as an instrument in this market.

This paper models the optimal services and incentives strategies of a personalization vendor whose objective is to maximize information acquisition in a market where consumers are heterogeneous in their privacy costs and this information is private to the consumer. The vendor has two broad options in this market; first he may offer no rewards or coupons but use the only available instrument, i.e., services to maximize his objective and second, the vendor can employ some form of incentives such as a coupon along with his services offering. We examine these possibilities through two mechanism designs where in one, the vendor engages the market through a fixed offering (either services alone or a fixed services-coupon contract) and in the other he offers an incentive-compatible schedule that comprises of either services alone or a menu of services and coupons.

1.2 Review of relevant literature

In examining prior work on incentive design, there is not any specific stream of research that has studied mechanism design for NFD goods, with some exceptions in computational approaches to auction design (Sandholm, et al. 2002). With regards to information goods and services, there is a vast literature in marketing and information systems that have studied markets from a pricing perspective. For example, there are many groups of researchers who have studied pricing of databases (Jain and Kannan 2002, Stonebraker, et al. 1996), application service providers (Cheng and Koehler 2003), and digital or information goods (Bakos and Brynjolfsson 1999, Varian 1997). From the point of view of mechanism design for information services where price is an important strategic variable, Mendelson and Whang (1990) develop an incentive compatible usage-based pricing scheme for various classes of users, which has been extended to non-linear pricing contracts based on delay and capacity costs (Dewan and Mendelson 1990). Gupta et al. (1997) model Internet pricing through a discrete approach, where the optimal strategy is based on a number of dimensions including service execution, quality of service and usage. Generally usage-based pricing is considered to be better than fixed offerings (Maskin and Riley 1984) although recent work by Sundararajan (2004), investigates this superiority and observes that a unique feature of information goods, namely zero marginal cost, can lead to alternative optimal pricing strategies. He finds that a combination of variable and fixed fee contracts may be optimal for this information goods market when there is marginal cost of administering usage based pricing. Even though our non-price market does not suffer from any form of marginal costs (including production, administration and delivery), there are some structural similarities between the above work and ours, including the zero marginal cost of production. At a later point in this paper we shall examine the impact of this marginal cost of administration on the optimality of the non-linear contract.

Indeed incentive-compatible mechanisms are not only designed for markets involving prices but they are often employed in many other contexts as well; e.g., there is large set of literature in economics on optimal taxation policies, labor contracts and utility regulation where the nature of the interaction between a principal and agents are based on non-price variables (See Salanié (1997) and Laffont and Martimort (2002) for a full discussion). One such work in management that is of relevance to us is a policy-maker's decision choice between a lump-sum and an output-based subsidy for incentivizing pollution-creating firms to adopt newer technologies, where the firms' operating conditions are unknown to the regulator (Levi and Nault 2004); in their model the principal and agents are the regulator and firms respectively and the firms' profit functions are non-monotonic concave in output similar to our agent's (the consumer) utility in services consumed. Thus while the context is different, there are some similarities in the model setup even if our approach to the mechanism design and analyses are quite different. However, their finding that under certain conditions a simple lump-sum subsidy emerges superior to an output based one, for firms to switch to a cleaner technology, is somewhat surprising and needs to be re-examined. If we were to extend this result to our context, it would appear that a vendor might perhaps be better off through fixed services-coupon offering than if he were to offer a variable usage based menu of coupons to the market.

While the development of incentive-compatible designs employed in prior work serve as a useful point of comparison, there are some important aspects of our market characterization that need to be noted:

i. NFD property of the good, where consumers have optimal satiation levels and thus reducing efficacy of buffet offerings of all services. This characterization is in contrast to earlier works on non-linear contracts, where consumers' utility is commonly monotonic and non-decreasing even if concave.

ii. Zero marginal cost of serving consumers under both fixed and variable contracts as in the case of personalization there are no marginal costs of offering multiple versions of a toolbar to multiple people, once it is created.

iii. Unavailability of pricing instruments to a vendor in this market, where an option of rewards/coupons as an instrument for both fixed and variable contracts need to be explored.

The paper is organized as follows: In §2, we provide the basic model set up including the consumers' functions and the vendor's objective functions; and derive optimal contracts under full information as a benchmark. In §3, we examine different options including fixed vs. variable offering and the provision of incentives for each of those cases, under information asymmetry. We conclude this section by developing a bunched solution as the optimal contracting mechanism. In §4, we discuss our results through numerical illustrations of 3 different markets and identify suitable theoretical and managerial recommendations.

2 Model

Our model consists of a *principal* – that is an online portal or a vendor who provides free personalization services and *agents* – who are consumers of these services. Consumers engage in a privacy calculus in their decision to use personalization services as they incur privacy costs in sharing information needed for this activity (Culnan and Armstrong 1999). This willingness to share information is based on the consumer's perceived benefits of disclosure balanced with its risks (Derlega, et al. 1993). Consumer behavior in this context has been modeled by prior research (Chellappa and Shivendu 2006) as a function of consumers' marginal value for personalization p and their coefficient of information privacy concerns r given by $u(s,p,r,i) = ps - ri^2$, where is s is the level of personalization services and i is the preference information shared by the consumer.

The number of personalized services that can be created from a unit of information is commonly a function of the prevalent personalization and data mining technologies (Raghu, et al. 2001, Winer 2001). One can view this as a production function wherein a technology ψ determines how many services (s) can be created from some information (i) that is provided. This technology relationship is represented by $s = \psi^{-1} i$, where the information and services are ordered to be increasingly personal (Chellappa and Shivendu 2003). An advanced technology $(\psi < 1)$ would imply that many personalization services can be created from one unit of preference information, although it is generally accepted that this technology $(\psi \geq 1)$ is still evolving despite significant advances in information acquisition (Chen and Hitt 2002). Since the vendor determines the number of personalization services to be offered in the market, and through the usage of services the consumers determine how much information they will share, we could write the consumer's utility as a function of personalization services consumed

$$u(s, p, r, \psi) = ps - r(\psi s)^2 \tag{1}$$

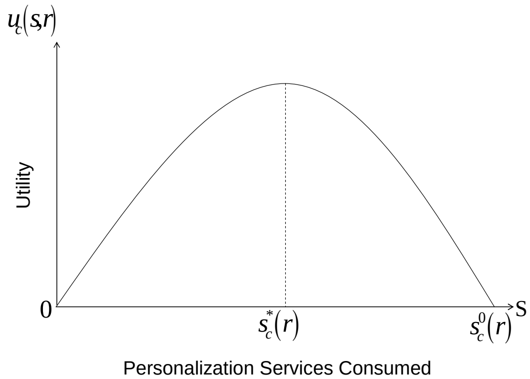

Figure 1: Consumer's utility for an NFD good (personalization services)

It is important to note two salient aspects of consumption here: First, there is no price to be paid to a vendor in consuming these services, they are free. Second, more services are not necessarily better and indeed the consumer has an optimal service level $s_c^* = \arg\max_s u(s, p, r, \psi)$ that he prefers over all other service levels. The latter (and its quadratic form) is a characteristic of a class of economic goods with the "no free disposal" property (Nahata, et al. 2003). Although somewhat less studied than goods with free disposal, conceptually, no-free disposal implies that individuals derive disutility from consuming additional quantities beyond their satiation level, e.g., hikers prefer to carry an optimal sized water container rather than very small or large bottles. We can observe that the utility function is non-monotonic (an inverted-U function) in services consumed, is also characterized by two levels s_c^* and s_c^0, the utility maximizing and break-even service $\left(s_c^0 : u(s, p, r, \psi)\,|_{s=s_c^0} = 0\right)$ levels respectively.

The consumers in the market are assumed to be homogeneous in their marginal value for personalization but heterogeneous in their concerns for privacy such that the co-efficient of privacy costs $r \in [\underline{R}, \overline{R}]$ is distributed with density function $f(.)$ and cumulative density $F(.)$ that is continuously differentiable. Thus the most privacy-sensitive consumer or privacy-seeker has $r \to \overline{R}$, while the least privacy sensitive consumer or convenience-seeker is given by the lower bound $(r \to \underline{R})$. Further, $f(.)$ is assumed to be single-peaked (uni-modal) and is everywhere positive on its support such that its hazard function $h(.) = \frac{f(.)}{F(.)}$, satisfies the monotone hazard rate property, i.e., $h'(.) \leq 0$. The assumptions on the distribution are quite standard and most commonly known distributions satisfy the above properties.

2.1 Vendor

Our vendor is a monopolist who has marginal value for information given by α, and has to determine the optimal number of services to offer with coupons if any. He can possibly offer a take-it or leave-it contract (a toolbar of fixed length) where he offers one service level to the entire market or variable services (toolbars of multiple lengths), where each consumer self-selects the service level optimal for them. Similar to Sundararajan (2004), we investigate the strategies of a capacity unconstrained vendor, i.e., he can offer as many services as the market can consume. Thus the vendor's problem is one of information acquisition (revenue maximization) and given by $\pi = \max_s \alpha A(s,p,i)$, where $A(s,p,i)$ represents the aggregate information acquired by the vendor in the market for s services offered. For some personalization technology ψ, this objective function can be written as

$$\begin{aligned} &\pi = \max_s \alpha A(s,p,\psi) \\ &\text{s.t.} \quad u(s,p,r,\psi) \geq 0 \end{aligned} \tag{2}$$

Along the lines of many information goods models, there are no marginal costs of production or usage of services, however there are two additional nuances to model setup that are unique to information services such as personalization. First, unlike his model we do not assume any costs of administering services, since once a toolbar is downloaded and embedded by a consumer it automatically populates the vendor's database with consumer information acquired. Second, versioning is costless for these toolbars, i.e. once a toolbar of a given length (characterizing the maximum number of services) is created, it is costless to create versions (toolbars of multiple lengths) that offer only a subset of the original. First we shall consider the objective function when the vendor has only one instrument at his disposal, i.e., personalization services. If the vendor chooses a fixed services strategy, the objective function in equation (2) can be written as

$$\pi_{fs-nc} = \int_{\underline{R}}^{r_{fs-nc}} (\alpha\psi s) f(r) dr \qquad \text{s.t.} \quad u(s,p,r,\psi) = ps - \psi^2 r s^2 \geq 0 \,|_{r \in [\underline{R}, r_{fs-nc}]} \tag{3}$$

And for variable services strategy it can be written as

$$\pi_{vs-nc} = \int_{\underline{R}}^{r_{vs-nc}} \alpha\psi s(r) f(r) dr \quad \text{s.t.} \quad u(s,p,r,\psi) = ps - \psi^2 r s^2 \geq 0 \,|_{r\in[\underline{R}, r_{vs-nc}]} \tag{4}$$

As discussed earlier, use of an additional instrument is generally helpful to a vendor in a market with asymmetric information and hence the vendor can explore an option of providing a side-payment or reward in the form of coupons for consumers that download and use his personalization services and hence provide information. Let the side-payment such as through a coupon or any other numeraire good be given by t. In designing a fixed-strategy, the vendor will offer some service-coupon pair $\{s,t\}$ for the entire market, while in the variable case, he will offer a menu of service-coupon pairs $\{s(r), t(r)\}$. We can thus re-write the objective function of the vendor in each case as

$$\pi_{fs-fc} = \int_{\underline{R}}^{r_{fs-fc}} (\alpha\psi s - t) f(r) dr \quad \text{s.t.} \; U(u(s,p,r,\psi),t) = ps - \psi^2 r s^2 + t \geq 0 \,|_{r\in[\underline{R}, r_{fs-fc}]} \tag{5}$$

and, where the capital letter $U(\ )$, represents the consumers total utility from using personalization services as well as obtaining the incentive

$$\begin{aligned} \pi_{vs-vc} &= \int_{\underline{R}}^{r_{vs-vc}} (\alpha\psi s(r) - t(r)) f(r) dr \\ \text{s.t.} \quad & U(u(s(r),p,r,\psi),t(r)) = ps(r) - \psi^2 r s^2(r) + t(r) \geq 0 \,|_{r\in[\underline{R}, r_{vs-vc}]} \end{aligned} \tag{6}$$

Note that throughout the paper, we shall use subscripts *fs* or *vs* for representing fixed or variable service offerings hyphenated with *nc*, *fc*, or *vc* for no-coupon, fixed-coupon or variable-coupon offerings. Before we analyze vendor strategies under information asymmetry, we shall first consider a benchmark case assuming full information and there we use capital notations such as Π, S, T and FS-FC.

2.2 Full information contracts

When a monopolist knows the privacy sensitivities of each individual in the market, he will attempt to extract full-surplus from each of them. Since the vendor's objective function is strictly increasing in services used and there are no marginal costs to him, it is easy to see that the vendor will set the individual rationality (IR) constraint to be binding for each individual and everyone will be required to use their respective break-even level of services $s_c^0(r)$.

Lemma 1 (VS-NC & VS-VC): *Under full information, the optimal services strategy when no reward is offered (VS-NC) will be given by* $S^*_{VS-NC}(r) = \dfrac{p}{\psi^2 r}$ *and the services strategy with* *rewards (VS-VC) is given by the pair* $S^*_{VS-VC}(r)=\dfrac{\alpha\psi+p}{2\psi^2 r}$, $T^*_{VS-VC}(r)=\dfrac{(\alpha\psi+p)(\alpha\psi-p)}{4\psi^2 r}$.

In both cases the vendor will serve the entire market given by $r\in\left[\underline{R},\overline{R}\right]$*, where the profits are given by* $\Pi^*_{VS-NC}=\dfrac{\alpha p}{\psi}\int_{\underline{R}}^{\overline{R}}\dfrac{f(r)}{r}dr$ *and* $\Pi^*_{VS-VC}=\left(\dfrac{\alpha\psi+p}{2\psi}\right)^2\int_{\underline{R}}^{\overline{R}}\dfrac{f(r)}{r}dr$ *respectively.* ■

First note that, under full information, it is never optimal for the vendor to offer a fixed-service and/or a fixed-coupon contract. The basic intuition behind this observation is quite straight-forward in that a monopolist under full information will always engage in first-degree discrimination. Furthermore, the vendor will always find it optimal to engage in couponing if and only if $\alpha\psi>p$, i.e., only if the vendor's marginal value of services used by the consumer is greater than the consumers' own marginal value parameter. This observation is derived from the condition that in order for the vendor to engage in couponing, $T^*_{VS-VC}(r)>0$ which implies $\alpha\psi>p$, and comparing profits we can see that $\Pi^*_{VS-VC}>\Pi^*_{VS-NC}$ when $\alpha\psi>p$. We can also see that the consumer surplus (with or without couponing) is zero, and the social welfare is the producer surplus. It can also be observed that as personalization technology improves (ψ decreases), the service level offered to the consumers and the profit to the vendor increases, while the coupons provided to the consumers is decreasing. However, note that the consumer welfare (remains zero) will not change with improvements in technology and as long as the vendor is not capacity constrained, he will always extract full surplus from each consumer in a zero-marginal cost setup.

SYMBOL	DEFINITION	SYMBOL	DEFINITION
p	Consumer's marginal value for personalization services	$r\in\left[\underline{R},\overline{R}\right]$	Consumer privacy cost coefficient – distributed with pdf $f(.)$ and cdf $F(.)$
α	Vendor's marginal value for information	t	Incentives
s	Personalization services (s_c^* - surplus maximizing service level; s_c^0 - break-even service level)	ψ	Technology that determines how many personalized services can be created for a unit of information ($i: i=\psi s$)
$u(p,r,\psi,s)$	Consumers' utility from personalization services	$U(p,r,\psi,s,t)$	Consumers' utility from personalization services and incentives
$\{s,t\}$	Fixed contracts	$\{s(r),t(r)\}$	Variable contracts

Table 1: Key Notation

It is perhaps more useful to analyze vendor strategies under information asymmetry since privacy costs of consumers are private information and often the vendor has no way of knowing which particular consumer is a privacy-seeker and which one is a convenience-seeker. In §3 we develop vendor contracts under information asymmetry while keeping in mind that any such optimal contract has to be one where consumers will self-select.

3 Vendor Strategies under Information Asymmetry

Under information asymmetry, the vendor does not have knowledge of individual privacy sensitivities and only has information on their distribution f. While it is quite straight forward that he will not offer fixed-services contracts under full information, under asymmetry both fixed and variable services contracts with and without coupons are viable strategies. In the following section we shall examine the optimal fixed-services contracts with and without couponing strategies while in §3.2 we will investigate the variable-services offering.

3.1 Fixed services contracts

Lemma 2 (fs-nc): *Under information asymmetry, the optimal fixed-services contract in the absence of any rewards will be given by* $s^*_{fs-nc} = \frac{p}{\psi^2} h\left(r^*_{fs-nc}\right)$. *The vendor will only serve a portion of the market given by* $r \in \left[\underline{R}, r^*_{fs-nc}\right]$, *where the profits will be* $\pi^*_{fs-nc} = \frac{\alpha p}{\psi} f\left(r^*_{fs-nc}\right)$. ■

When the vendor offers a fixed-services contract to the entire market, all consumers that derive positive utility for that service level will contract with the vendor. Let r_{fs-nc} be consumer who is indifferent between taking the contract and not participating, since $\frac{\partial u(r,s)}{\partial r} < 0 \,|_{\forall s>0}$, we can safely conclude that all consumers whose privacy sensitivity is *less* than $r_{fs-nc} = \frac{p}{\psi^2 s}$ will individually contract, i.e., the market given by $r \in \left[\underline{R}, r_{fs-nc}\right]$ will be served. From the optimal solution we can observe the following about the portion of the market served under fixed contracts: i) $r^*_{fs-nc} \geq$ mode of $f(.)$ - implying that for any symmetric unimodal distribution a fixed-services contract will serve more than half of the market potentially leaving out some highly privacy-sensitive consumers. Also note that the portion of the market served will be *smaller* if the market is predominantly made up of convenience seekers (right skewed distribution), than one comprising largely of privacy seekers (left skewed distribution) though the profits to the vendor are higher in the former. ii) We can also see that as the personalization technology improves (ψ decreases), the service levels, market-served and the vendor profits are all increasing.

This contract is analogous to a buffet-pricing or fixed-fee pricing under asymmetry that is common in telecommunications and other literature (Laffont and Martimort 2002). Note that in both in our contract and the buffet-price contract, only a portion of the market is served, albeit for different reasons. Commonly in price contracts for free disposal goods/services, even if all consumers will derive positive value for usage of services, the prices charged may eliminate some consumers in the market due to differences in their reservation prices. In our model, there are no prices but the NFD property of the good is the source of some consumers being eliminated from the market served, i.e., there may be consumers for whom the offered service level is more than their break-even service level. Another interesting observation from Lemma 2 is the fact that even when the vendor has no costs of serving consumers, it is not optimal for him to offer the entire set of feasible services. The intuition behind this is that the due to the NFD property of the services, the vendor is capped by the maximum service that can be consumed in this market, given by the least privacy-sensitive consumer's break-even level $\left(s_c^0(\underline{R}) = \dfrac{p}{\psi^2 \underline{R}}\right)$. From any one consumer this level represents the maximum surplus that can be extracted and in determining his optimal offering he considers the tradeoff between reducing this service level (thus reducing surplus (ψs) extracted from each consumer) and increasing the participating market size (only one consumer will participate at $s_c^0(\underline{R})$).

The full information model suggests that if a vendor's marginal value for information from services consumed, is higher than that of the consumers' then he should offer incentives along with services. We now explore this option under asymmetry as well given that an additional instrument to vendor is often beneficial in such markets. Let the vendor offer a fixed incentive along with the fixed-services contract such that a consumer who uses a fixed level s_{fs-fc}, will also get an incentive t_{fs-fc}. Let r_{fs-fc} be consumer who is indifferent between taking the fixed-service/fixed-coupon contract and not participating. All consumers whose privacy sensitivity is less than $r_{fs-fc} = \dfrac{ps_{fs-fc} + t_{fs-fc}}{\psi^2 s_{fs-fc}^2}$ will individually contract, i.e., the market given by $r \in [\underline{R}, r_{fs-fc}]$ will be served and hence the vendor's objective function will be given by (5).

Lemma 3 (fs-fc): *Under information asymmetry, the optimal fixed-services contract with complementary couponing strategy is given by* $s^*_{fs-fc} = \dfrac{p+\alpha\psi}{2\psi^2} h\left(r^*_{fs-fc}\right)$. *The vendor will offer coupons given by* $t^*_{fs-fc} = \dfrac{(\alpha\psi - p)\, s^*_{fs-fc}}{2}$ *and will serve a portion of the market* $r \in \left[\underline{R}, r^*_{fs-fc}\right]$ *where his profits will be given by* $\pi^*_{fs-fc} = \left(\dfrac{\alpha\psi + p}{2\psi}\right)^2 f\left(r^*_{fs-fc}\right)$. ■

This Lemma leads to the following proposition.

Proposition 1: *Couponing is an optimal strategy only for a vendor with sufficiently high MVI* $(\alpha\psi > p)$. *In the fixed services contract, while couponing always improves vendor profits, consumer surplus and social welfare, the market size served remains the same*$\left(r^*_{fs-nc} = r^*_{fs-fc}\right)$. ■

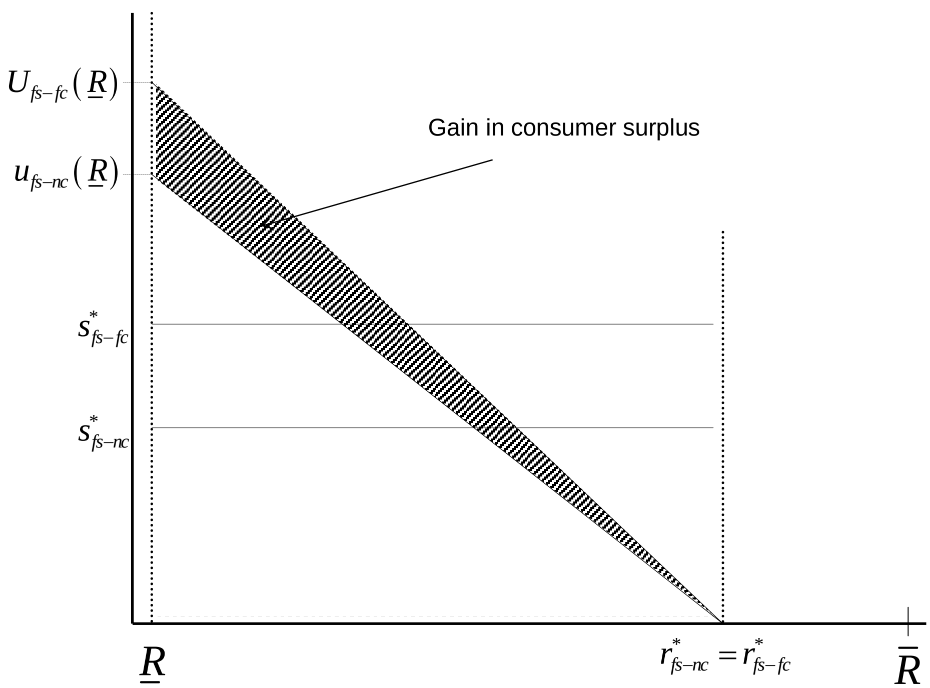

Figure 2: Gains from couponing

Once again we see from Lemma 3, that the vendor does not find it optimal to serve the full market. Intuitively it might appear that providing additional incentives will lead to participation from more consumers than those observed in Lemma 2. However, counter to intuition, we can see from Proposition 1 that the vendor's optimal strategy is to serve the same market as in the absence of any incentives. The economic rationale behind this observation is as follows: A fixed-coupon essentially increases the break-even service level for all consumers in the market; the vendor has one of two options to extract this surplus. One, he could offer the same service level as in Lemma 2, and thus get a larger portion of the market to participate. Two, he could consider increasing the service level for the same market and compensating each consumer for the disutility incurred such that that the last consumer (of type r^*_{fs-nc}) still gets zero utility. The decision between choosing options one and two will depend upon the vendor's marginal gain from additional services versus the marginal gain from increased market size. From Lemma 2, we already know the optimal market size as a function of the consumer distribution and since the fixed-coupon only changes the intercept and does not affect this tradeoff discussed in Lemma 2, we can see that the vendor will opt for increased services and compensate each consumer with a fixed-coupon.

Comparing Lemmas 2 and 3, we can see that the producer surplus is higher with couponing for the obvious reason that the marginal value from the services consumed is higher for the producer as compared to the consumers in the market $(\alpha\psi > p)$ and he can provide back a portion of this surplus to each consumer as an incentive and still be left with positive gains. Also when $\alpha\psi = p$, the service level in Lemma 3 converges to that in Lemma 2, and a coupon is not offered $\left(t^*_{fs-fc} = 0\right)$. The above observations are similar to the ones in the full information case described in Lemma 1. However note that in the case of full information the vendor was able to extract back the entire consumer surplus (from the incentive provided), while the information asymmetry prevents him from doing so in Lemma 3. Hence he is only able to extract full surplus from the last participating type and all others are left with a residual gain that cannot be extracted (as shown in Figure 1). These lead to increased consumer welfare, producer surplus and hence social welfare in the total market though the market size remains the same.

Once again, we note that in fixed-services contract even with zero marginal costs and no capacity-constraints, the vendor does not end up serving the entire market as some privacy-seekers are left un-served. This observation perhaps immediately merits the investigation of variable services contract under information asymmetry.

3.2 Variable services contracts with no rewards

In any market with consumer heterogeneity along some dimension, when variable contracts are offered the vendor has to keep in mind that consumers will self-select into different contracts. Since a truthful direct revelation mechanism is optimal under asymmetry, the vendor needs to construct one contract for each consumer type such that it is incentive compatible and hence truthful. In the case of variable services with no additional incentives, the vendor is restricted to using the service level as the only instrument. Since the vendor surplus is strictly increasing in services consumed, in the full information case even with one instrument the vendor developed a contract based on the consumer's break-even service level wherein he was able to extract full surplus. Under asymmetry this strategy is not viable for the simple reason that consumers will lie (if asked to declare their privacy type) and will self-select a service level lower than their break-even services. Thus the menu under full information will not be incentive compatible under asymmetry.

Lemma 4 (vs-nc): *Under information asymmetry, the optimal variable services contract in the absence of coupons is given by* $s_{vs-nc}^{*}(r)=\dfrac{p}{2\psi^2 r}$. *All consumers in the market will be served and the profits will be given by* $\pi_{fs-nc}^{*}=\dfrac{\alpha p}{2\psi}\int_{\underline{R}}^{\bar{R}}\dfrac{f(r)}{r}dr$. *A truth-revealing variable services contract in the absence of incentives will be consumer surplus maximizing* ■

The Spence-Mirrlees property or the constant sign assumption (Guesnerie and Laffont 1984) holds for the class of utility function in our market, i.e., $\dfrac{\partial^2 u}{\partial s \partial r}$ has a constant sign (negative in our case). Intuitively this condition states that for a given service level, the utility to a consumer of given privacy sensitivity is always higher than any other consumer with higher privacy costs (higher r). This suggests that it is possible to construct an incentive-compatible contract that is both locally and globally satisfying. When the vendor offers some service level and consumers are allowed to self-select then they will elect to use their optimum level s_c^* (if offered) and if the maximum service offered is less than their optimal, they will choose the maximum level $\left(\min\left\{s_c^*, s_{\max}\right\}\right)$. Since services are costless to the vendor and his profits are increasing in services, the maximum service level offered will be one that is optimal to the least privacy-sensitive consumer $\left(s_{vs-nc}^*(\underline{R})\right)$ and to all others the vendor will offer their respective optimum as given in Lemma 4. It is perhaps surprising to the reader that a monopolist finds consumer surplus maximizing contracts to be optimal. This result is unique to the NFD property of the good and unavailability of any other instrument such prices, and the need for the contracts to be truth-revealing. This result also is in direct contrast to the full information case where consumer surplus in the market was zero and vendor profits were twice that in

Lemma 4. Thus our analysis reveals the severity of information asymmetry in markets for NFD goods and the impending need for another instrument in the market.

Proposition 2: *It is never optimal for the vendor to offer a fixed-service, variable-coupon or a variable-service, fixed-coupon contract.* ■

When the impact of information asymmetry is severe, it becomes more important to seek another instrument of interaction in the market that the vendor can employ in a strategic fashion. In the fixed-services contracts, when rewards or incentives were introduced we only considered a fixed-services fixed-coupon contract, i.e., we did not consider the option of providing variable coupons along with fixed services. Proposition 2 negates the possibility of offering such a mixed contract where one instrument is fixed while the other is variable. This observation is not a function of the NFD property of the good or our market alone but is generally applicable to the use of multiple instruments in situations of information asymmetry. One can see that in price-quantity schedules vendors will set prices such that the consumer is incentivized to buy more, i.e., higher quantities will imply higher discounts or lower prices (the two move in opposite directions). Similarly in price-quality schedules, we can see that higher quality implies higher prices, i.e., the two move in the same direction; the fundamental requirement being that both instruments have to be related in that if one changes the other also has to. Otherwise the agents (consumers in our model) will always lie so as to maximize their returns on the variable instrument and the vendor will be unable to construct an optimal truth-revealing contract.

3.3 Designing variable-service variable-coupons contracts

In this section we introduce couponing or side-payments as an additional instrument to the variable-services contract discussed in §3.2. As illustrated by Proposition 2, a fixed option for coupons is not recommended and hence we explore the design of variable-services, variable-coupons contract. If such a menu can be constructed as a direct revelation mechanism, then it should indeed satisfy the truth-revelation principle. Thus for a type r declared by consumers, the vendor needs to offer a pair $\{s_{vs-vc}(r), t_{vs-vc}(r)\}$ such that the consumers will not lie.

Lemma 5 (vs-vc): *A general truth-revealing variable service-variable coupon menu under information asymmetry for the entire market* $r \in [\underline{R}, \overline{R}]$ *will be given by* $s_g^*(r) = \frac{\alpha\psi + p}{2\psi^2}\frac{1}{k(r)}$

and $t_g^*(r) = \frac{\alpha\psi + p}{2\psi^2}\left(\frac{-p}{k(r)} + \frac{(\alpha\psi + p)}{2}\left(\frac{r}{k^2(r)} + \int_r^{\bar{R}} \frac{dx}{k^2(r)}\right)\right)$, *where* $k(r) = \left(r + \frac{1}{h(r)}\right)^{-1}$ *subject to* $\dot{s}(r) \leq 0; \dot{t}(r) \leq 0; U(r) \geq 0$. ■

In deriving the optimal contracts, we need to satisfy both local and global incentive compatibility constraints. The former requires that the services menu is non-increasing in privacy costs, i.e., the incentive compatible contract should be such that higher r should be offered lower services $(\dot{s}(r) \leq 0)$ and the latter is ensured by the constant-sign or Spence Mirrlees crossing property. We can also see that the rent to each type has to be non-increasing in privacy types, i.e., $(\dot{U}(r) \leq 0)$ such that the most inefficient type (the most privacy-sensitive consumer $\bar{R}$) gets zero rent and also that the incentives are decreasing with privacy sensitivities $(\dot{t}(r) \leq 0)$. Note that while the development of incentive-compatible contracts described in Lemma 5 lays specific conditions on the slopes of the services and incentives menu and the information rent, it does not specifically require that incentives always have to be positive. For the class of utility functions we consider, in order to provide non-negative incentive-compatible rent to entire market, the menu of incentives might change from positive (coupons/rewards) to negative if some vendor and/or market conditions differ.

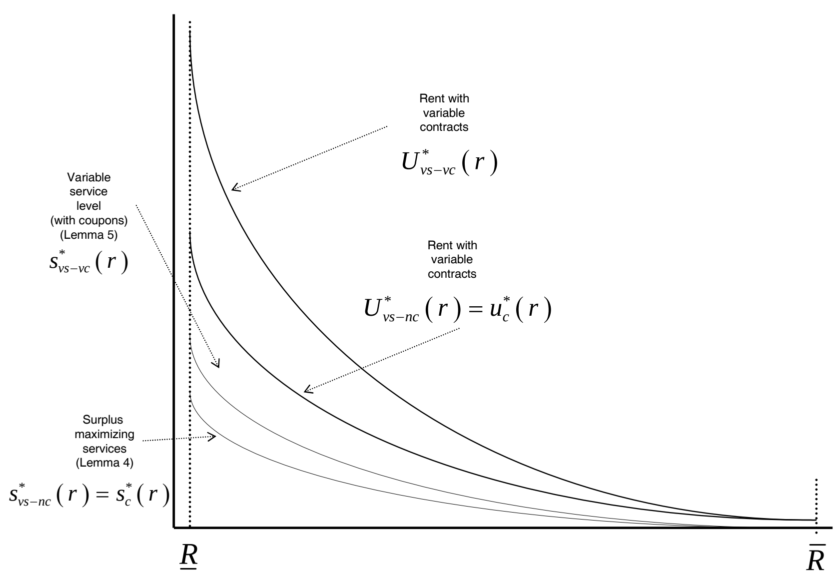

Figure 3: Service levels and rents under variable contracts

Corollary 1: *The menu of incentives* $t^*_{vs-vc}(r)$ *described in Lemma 5 is a) Always positive* $\left(t^*_{vs-vc}(r) > 0 \,|_{\forall r \in [\underline{R}, \bar{R}]}\right)$ *if* $\alpha\psi > p\left(1 + \frac{2}{\bar{R}f(\bar{R})}\right)$*; b) Always negative* $\left(t^*_{vs-vc}(r) < 0 \,|_{\forall r \in [\underline{R}, \bar{R}]}\right)$ *if* $\alpha\psi < p$*; and c) For vendors with MVI such that* $p\left(1 + \frac{2}{\bar{R}f(\bar{R})}\right) > \alpha\psi > p$*, the menu is positive for low* r *and turns negative with increasing* r*.* ■

In our discussions on fixed contracts with rewards, we observed that couponing (or positive incentives to the consumers) is an optimal strategy when the marginal value to vendor from the services consumed is higher than that for consumers in the market. As Corollary 1 tells us the feasibility of positive couponing is not only a function of the relative marginal valuation of the vendor and the consumers in the market, but it is also a function of distribution of consumer types in the market. In order for a vendor to engage in variable-services variable-couponing contract for the full market, the vendor's marginal value for services has to be higher than if he were considering a fixed-services fixed-coupon contract.

Consistent with intuition Corollary 1 suggest that vendor will employ couponing when consumers are largely privacy seekers so as to incentivize them to participate (i.e., when $f(\bar{R})$ is large). A common sense approach might perhaps suggest that more privacy-sensitive types should be paid higher incentives to consume services, however counter to intuition our results clearly show that an incentive-compatible contract will require that low privacy types be paid higher incentives. The simple economic reasoning behind this observation is the fact that if the vendor does not pay a higher rent to the low privacy types, he risks the possibility of these consumers lying and declaring themselves to be more privacy sensitive than they truly are. To that extent we see that rents are decreasing in increasing privacy costs even in the fixed-services fixed coupon contracts although the trend was linear, while here the rents are decreasing in a non-linear fashion.

It is important re-iterate here that the results from Lemma 5 cannot be taken at face-value as no negative-incentives, i.e., prices are feasible in this market. If prices were indeed feasible, then the vendor would simply offer coupons to some, price some others or do one or the other for the whole market. Thus, from Corollary 1, we can conclude that if the vendor employs a variable-services variable-coupon contract and the market distribution and relative vendor characteristics are given by case (a), he will offer to coupons to everyone in the market and if defined by case (c) he will not engage in couponing at all. An interesting situation when the market and agents are not defined by either and are described by case (b); where in this case Corollary 1 would suggest that the vendor offer positive coupons to a portion of the market (as long as $t^*_{vs-vc}(r)$ is non-negative). However, this leaves the vendor with a strategy where the most inefficient type he serves is left with a positive surplus thus suggesting the possibility of a better solution than employing Lemma 5 for part of the population.

The personalization services are defined to be "free," i.e., the vendor will not be able to charge for the services. Let the consumer with zero incentives from the contract in Lemma 5 be given by $r_0 : t^*_{vs-vc}(r_0) = 0$. One option is to offer this consumer's variable services contract $s^*_{vs-vc}(r_0)$, to the rest of the consumers with higher privacy sensitivity $\left(r \in \left[r_0, \overline{R}\right]\right)$ with no additional incentives. This will indeed not alter the truth revelation principle of the contract. We can clearly see that this hints towards a bunched solution where vendor ends up offering $\left\{s^*_{vs-vc}(r), t^*_{vs-vc}(r)\right\} \forall r \in [\underline{R}, r_0]$ and $\left\{s^*_{vs-vc}(r_0), t^*_{vs-vc}(r_0)\right\} \forall r \in [r_0, \overline{r}]$, where $U(\overline{r}) = u(\overline{r}) = 0$ such that consumer types of $r \in \left[\overline{r}, \overline{R}\right]$ are not served. Note that such a contract is a slice of the variable contract derived in Lemma 5 where applying external constraint $t^*_{vs-vc}(.) \geq 0$ dictates what portions of the market will be served and how. Incorporating this restriction in the optimization of the problem itself will clearly lead to a result that cannot make the vendor worse off and perhaps better.

Lemma 6 (vf – bunched solution): *When the vendor faces a market defined by case (c) in Corollary 1, the optimal contact is a bunched solution where he will offer the variable contract* $\left\{s^*_{vs-vc}(r), t^*_{vs-vc}(r)\right\}$ *from Lemma 5 for* $r \in \left[\underline{R}, r^*_{vf}\right]$*, a fixed contract* $\left\{s^*_{vs-vc}\left(r^*_{vf}\right), t^*_{vs-vc}\left(r^*_{vf}\right)\right\}$ *from for* $r \in \left[r^*_{vf}, \overline{r}^*_f\right]$*, and consumers with the highest privacy costs (defined by* $r \in \left[\overline{r}^*_f, \overline{R}\right]$ *will not participate. The most privacy sensitive participating type will be given by* $\overline{r}^*_f = \dfrac{p s^*_{vf}\left(r^*_{vf}\right) + t^*_{vf}\left(r^*_{vf}\right)}{\psi^2\left(s^*_{vf}\left(r^*_{vf}\right)\right)^2}$ *where* r^*_{vf} *is the solution to equation (7).* ■

The constraint $t^*_{vs-vc}(.) \geq 0$ for the entire market implies that $\dot{t}^*_{vs-vc} = 0$ for a portion of the menu of contracts since otherwise Lemma 5 will require charging some consumers. This further implies (due to global incentive-compatibility requirements on the information rent $\dot{U}(r)$) that the monotonicity constraint on the services menu (that $\dot{s}^*_{vs-vc}(r) \leq 0$ - equation (15) in the Appendix) is binding, i.e., $\dot{s}^*_{vs-vc}(r) = 0$ for a portion of the market. Whenever monotonicity constraint is binding, Laffont and Martimort (2002) and other works on contract theory have suggested the optimality of a bunched solution.

In order to derive the formal bunched solution, let the vendor offer a variable services contract $\{s_{vf}(r), t_{vf}(r)\}$ to consumers defined by $r \in [\underline{R},...,r_{vf}]$, i.e., $(\dot{s}_{vf}(r) < 0, \dot{t}_{vf}(r) < 0)$ and provide a bunched contract $\{s_{vf}(r_{vf}^*), t_{vf}(r_{vf}^*)\}$ to serve consumers defined by $r \in [r_{vf},...,\overline{r}_f]$, i.e., $((\dot{s}_{vf}(r) = 0, \dot{t}_{vf}(r) = 0))$, where r_{vf}^* is the solution to:

$$r_{vf}^* = \arg\max_{r_{vf}} \int_{\underline{R}}^{r_{vf}} (\alpha\psi s(r) - t(r)) f(r) dr + (\alpha\psi s(r_{vf}) - t(r_{vf})) \int_{r_{vf}}^{\overline{r}_f^*} f(r) dr \tag{7}$$

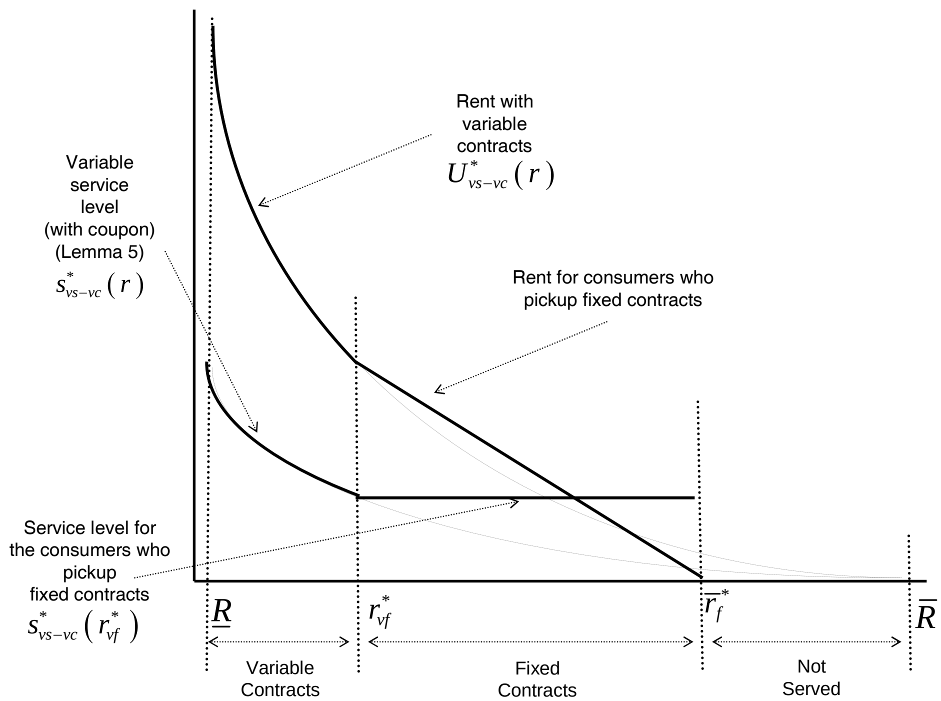

Figure 4: Bunched solution

3.4 Optimality of fixed and variable contracts for NFD goods with and without the option of providing incentives

Having developed various contractual mechanism designs, it is perhaps important now to recommend the optimal strategy for a vendor facing a market rife with heterogeneity and where he cannot possess full information on the customer characteristics. Our closed form results clearly show that whether a second instrument such as couponing can be employed or not is a function of the vendor's relative value for information from the services consumed. So we specifically discuss what the monopolist's preferred approach should be when he can and cannot engage the consumers through a second instrument.

Strategy	Service, Coupon	Market Served	Profits
VS-NC	$S^*_{VS-NC}(r) = \frac{p}{\psi^2 r}$	$[\underline{R}, \overline{R}]$	$\Pi^*_{VS-NC} = \frac{\alpha p}{\psi} \int_{\underline{R}}^{\overline{R}} \frac{f(r)}{r} dr$
VS-VC	$S^*_{VS-VC}(r) = \frac{\alpha\psi + p}{2\psi^2 r}$ $T^*_{VS-VC}(r) = \frac{(\alpha\psi + p)(\alpha\psi - p)}{4\psi^2 r}$	$[\underline{R}, \overline{R}]$	$\Pi^*_{VS-VC} = \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_{\underline{R}}^{\overline{R}} \frac{f(r)}{r} dr$
fs-nc	$s^*_{fs-nc} = \frac{p}{\psi^2} h\left(r^*_{fs-nc}\right)$	$[\underline{R}, r^*_{fs-nc}]$ $r^*_{fs-nc} = \frac{p}{\psi^2 s^*_{fs-nc}}$	$\pi^*_{fs-nc} = \frac{\alpha p}{\psi} f\left(r^*_{fs-nc}\right)$
fs-fc	$s^*_{fs-fc} = \frac{\alpha\psi + p}{2\psi^2} h\left(r^*_{fs-fc}\right)$ $t^*_{fs-fc} = \frac{(\alpha\psi - p) s^*_{fs-fc}}{2}$	$[\underline{R}, r^*_{fs-fc}]$ $r^*_{fs-fc} = \frac{\alpha\psi + p}{2\psi^2 s^*_{fs-fc}}$	$\pi^*_{fs-fc} = \left(\frac{\alpha\psi + p}{2\psi}\right)^2 f\left(r^*_{fs-fc}\right)$
vs-nc	$s^*_{vs-nc}(r) = \frac{p}{2\psi^2 r}$	$[\underline{R}, \overline{R}]$	$\pi^*_{fs-nc} = \frac{\alpha p}{2\psi} \int_{\underline{R}}^{\overline{R}} \frac{f(r)}{r} dr$
vs-vc vf	$s^*_{vs-vc}(r) = \frac{\alpha\psi + p}{2\psi^2} \frac{1}{k(r)}$ $t^*_{vs-vc}(r) = \frac{\alpha\psi + p}{2\psi^2} \left(\frac{-p}{k(r)} + \frac{(\alpha\psi}{} \right. \left(\left(\frac{r}{k^2(r)} + \int_{r}^{\overline{R}} \right. \right.$ $k(r) = \frac{1}{\left(r + \frac{1}{h(r)}\right)}$	$[\underline{R}, r^*_{vf}]$ $r^*_{vf} = \arg\max_{r_{vf}}$ $\int_{\underline{R}}^{r_{vf}} (\alpha\psi s(r) - t(r)) f(r$ $+\left(\alpha\psi s\left(r_{vf}\right) - t\left(r_{vf}\right)\right) \int_{r_{vf}}^{\overline{r}^*_f} f$	$\pi^*_{vf} = \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_{\underline{R}}^{r^*_{vf}} \frac{dr}{(rf(r) + F(r)}$ $+\left(\alpha\psi s^*_{vf}\left(r^*_{vf}\right) - t^*_{vf}\left(r^*_{vf}\right)\right)$ $\left(F\left(\frac{p s^*_{vf}\left(r^*_{vf}\right) + t^*_{vf}\left(r^*_{vf}\right)}{\psi^2 \left(s^*_{vf}\left(r^*_{vf}\right)\right)^2}\right) - F\left(r^*_{vf}\right)\right)$
	$s\left(r^*_{vf}\right)$ $t\left(r^*_{vf}\right)$	$[r^*_{vf}, \overline{r}^*_f]$ $\overline{r}^*_f = \frac{p s^*_{vf}\left(r^*_{vf}\right) + t^*_{vf}\left(r^*_{vf}\right)}{\psi^2 \left(s^*_{vf}\left(r^*_{vf}\right)\right)^2}$	

Table 2: Summary of results

Proposition 3: *When couponing is <u>not feasible</u> the optimality of fixed versus variable contracts will depend on the distribution of consumer types in the market. However, when couponing is* *feasible, the bunched solution described in Lemma 6 is weakly superior to any purely fixed or purely variable contracts.* (Proof of this proposition is discussed below) ■

When the only available instrument is services, it is not obvious if the vendor should engage in fixed or variable contracts as there are advantages and disadvantages of both. First, a fixed contract provides the advantage that he may be forcing some consumers to use services that are more than their optimal but in trying to maximize the portion of the market served he also ends offering less services than he could have for some consumers (convenience seekers with low r). The variable services-only contract results in a consumer surplus maximizing contract and commonly in any durable goods market it is perhaps rare if not impossible that a consumer surplus maximizing solution is also optimal from a monopolist's perspective. On the other hand for this market of NFD goods, there are situations where such a consumer-welfare maximizing contract can be a better mechanism. This intuition behind this observation is that when consumers are highly dispersed in their types, the fixed contract will end up serving a smaller portion of the market unless of course the service level is significantly lowered. In such a situation allowing consumers to use their own optimal level and ensuring that the whole market participates can indeed be a better option as compared to fixed contracts (either a low service level for a larger portion of the market or a high service level for a select few). Specific instances are discussed in the numerical example in §4.

Now consider the case when the vendor's marginal value for services consumed and the market distribution (conditions given in Lemmas 3, 5 and 6) is such that employing couponing as an additional instrument is indeed a feasible option. Proposition 3 tells us that a bunched solution, i.e., providing a fixed and variable contract is weakly superior to other strategies. The simple intuition behind this observation is that the purely variable and purely fixed contracts are two extreme cases of a market served by a bunched design. For a non-price market described in case (c) of Corollary 1, a bunched solution emerges superior to a purely variable contract design as the latter results in incentives being given to some consumers and prices charged to some others. For situation defined by case (a), when incentives are positive for the whole market, Lemma 6 will converge to Lemma 5. This is due to the fact that the variable contract designs are the same in both Lemmas 5 and 6 due to point-wise maximization of rent extraction and when $r_{vf}^* \to \overline{R}$ in Lemma 6, we see that no portion of the market will be offered fixed contracts. Similarly if $r_{vf}^* \to \underline{R}$, and $\overline{r}_f^* \to r_{fs-fc}^*$, the bunched solution will converge to the fixed contract described in Lemma 3. Thus a vendor cannot be worse-off by offering the bunched solution.

4 Discussion of the vendor strategies through numerical illustrations of different market characteristics

In the paper, we obtain general and closed-form solutions to vendor strategies. Given that we have not made any assumptions on the functional form of the distribution, it may not be easy to fully comprehend the results. Hence in this section we shall first discuss vendor strategies for differing market characteristics.

4.1 Numerical illustration

For the purpose of the numerical illustration, we assume that consumers' privacy costs have a general beta distribution given by $r \sim \text{Beta}(a,b), r \in [0,1]$. The general form of the Beta distribution is given by $f(r) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} r^{a-1}\left(1 - r^{b-1}\right)$, where $\Gamma(.)$ is the gamma-function. We represent different markets by varying the mean $\left(\mu = \frac{a}{a+b}\right)$ and variance $\left(\sigma^2 = \frac{ab}{(a+b)^2(a+b+1)}\right)$. We consider 3 different markets: A) Symmetric market (with $\mu = 0.5$), B) Market dominated by convenience-seekers (right-skewed with $\mu = 0.25$) and C) Market dominated by privacy-seekers (left-skewed with $\mu = 0.75$). We set the following values to other parameters as: $p = 2; \psi = 1; \alpha = 5$. Even for the same type of market (dominated by one consumer type or the other), the market may vary in the concentration of consumer types. Since it is not possible to analytically explore any comparative statics with respect to concentration of types in a market and since the efficacy of fixed and variable contracts can depend on this difference, we vary the Beta distribution parameters a and b such that the mean is kept constant while the variance changes for each of the 3 markets. When types are concentrated, the variance is low and when the types are dispersed, the variance is high.

Figure 5 illustrates vendor profits under fixed and variable services contract when offering incentives is not an option. If we had assumed a functional form for the distribution we would have derived a condition when the fixed contract is superior to the variable one and vice-versa. The numerical example clearly shows as variance increases, independent of market domination, fixed-contracts do badly. This is consistent with the general intuition that a one-size fits all approach is not recommended when consumers differ very much from each other. However also note that as the differences grow, the variable contract becomes more profitable to the monopolist even if they are consumer-welfare maximizing. This effect is pronounced in a market that is dominated by convenience seekers (B), since these consumer types by themselves prefer high number of services and the relative benefit from forcing consumer types to use more than their optimal (as a fixed contract would do) is small. Thus the strategy to be adopted will be a function of the distribution of consumer types in the market; although it is quite evident that independent of variance, a market dominated by convenience seekers (B) will fetch the highest profits for the vendor.

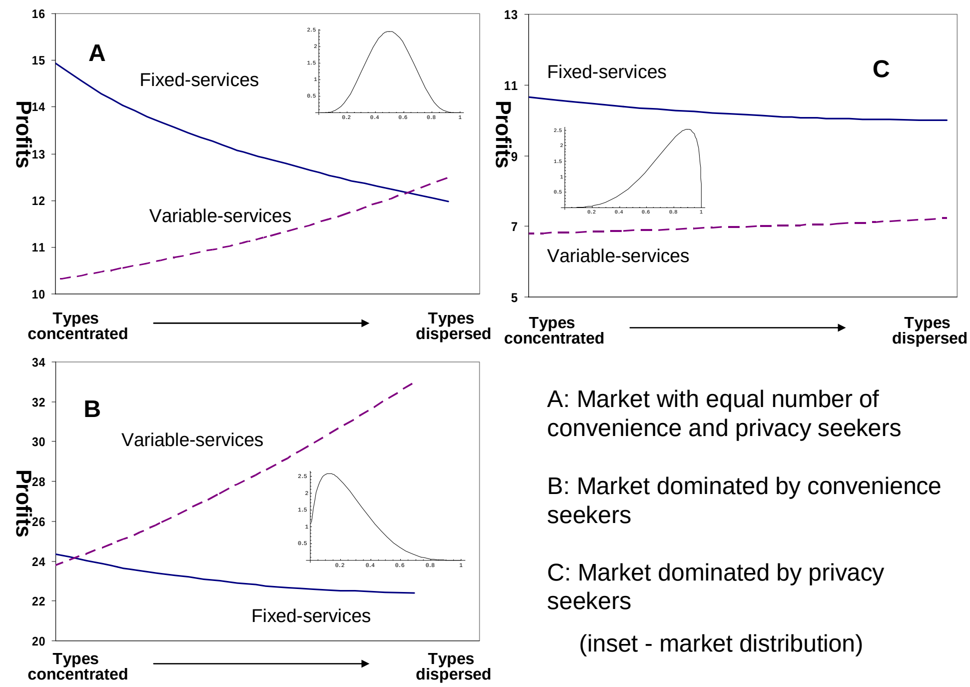

Figure 5: Dispersion of consumer types and vendor profits in the absence of couponing

Consistent with the discussion of Proposition 3, we can see that with the option of coupons, the bunched contract is always superior. Along the lines of earlier discussions on fixed contracts, increasing dispersion of types makes the fixed contract further worse-off and widens the gap between the two contracts. The NFD property of this good has certain interesting impacts on the nature of rent to be paid depending upon the "distance" of the neighbor's contract. For any consumer served by the variable design, the service contracted by the variable contract is greater than his surplus maximizing level. Hence, for a given consumer type $\underline{r}$, the surplus (from personalization services alone) is higher from a contract meant for a more privacy-sensitive neighbor $\overline{r}$. This surplus is first *increasing* in the "distance" between the two (surplus increasing as $\overline{r} - \underline{r}$ is increasing); however after a certain point (when the contract meant for $\overline{r}$ equals the surplus maximizing contract for $\underline{r}$) this surplus begins to *decrease*. Therefore the rent to be paid (purely as a function of surplus from services) to prevent $\underline{r}$ from picking up the lower contract is first increasing and then decreasing, which is the reason that incentives are first positive and then negative so as to ensure that the overall rents are decreasing in types (to ensure global incentive compatibility). But due to the constraint that prices cannot be charged, some of the types where incentives become negative are given a fixed contract. For a typical more-is-better good, rent is *strictly increasing* as a low type is more different than a high type (Mussa and Rosen 1978); and when the rent becomes so high because of extreme differences in the types or lack of enough low types to maintain the low segment, the shutdown condition comes into play (where only the high type is offered).

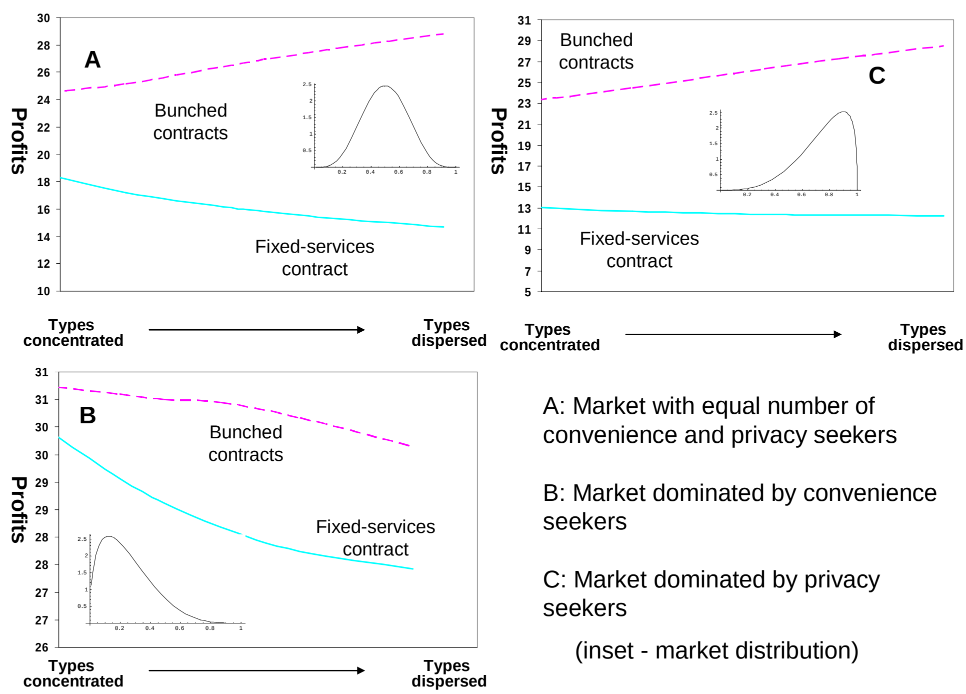

Figure 6: Dispersion of consumer types and vendor profits with couponing strategy

Similarly in our numerical example, in some cases (A and C), there is an optimal separation and enough privacy seekers to warrant a variable contract and with increasing variance and with no marginal cost of services, the separation amongst the consumer types leads to increase in the number of consumers served by both variable and fixed contracts, i.e., both r_{vf}^* and $\overline{r}_f^*$ are increasing. On the other hand, when a market is dominated by convenience seekers and the variance increases, there are not enough privacy-seekers to warrant rent to the convenience-seekers, and the fixed-portion of the bunched solution becomes increasingly prominent, i.e., r_{vf}^{*} and $\overline{r}_{f}^{*}$ are decreasing with variance; and as discussed earlier, fixed contracts perform poorly with increasing variance in the market. Although note that the profits are higher when the market is concentrated and dominated with convenience seekers (as in case B).

4.2 Discussion of results

From a theoretical point of view this work contributes to research in the area of information goods and services, particularly with regards to mechanism design. Recent research (Sundararajan 2004) observes that the zero marginal cost of producing digital goods *along with* some marginal cost in administering them, results in a combination of fixed and usage-based contracts being optimal. This work also points out (p. 1670) that if the marginal cost component does not exist, then a fully-revealing, purely usage-based contract becomes optimal along the lines of well known results of Maskin and Riley (1984) and others. However, our results point out that even when there are no marginal costs to the vendor, a bunched solution or a combination of fixed and variable contracts is always optimal. The bunched solution subsumes both the fixed and variable contracts and the optimality of the solution is due to the NFD nature of the services and non-price market characteristics. The economic intuition behind this result is that a high-type consumer's surplus from lying is non-monotonically increasing in services resulting in negative incentives and the non-price constraint in the market forces the vendor to consider fixed contracts.

There are structural similarities between the principal-agent interactions in our model and that of regulator-firm models where firms create pollution or environmental damage to the society. In the Levi and Nault's (2004) model either a subsidy is offered for firms that convert to non-polluting technology or a tax is levied on production, where the firm profits are non-monotonic concave in output. Further, they impose a constraint that subsidies (taxes) cannot be negative (positive), i.e., the market doesn't allow a contract where a portion of the firms get taxed and the better-technology firms receive subsidies. The authors then conclude that under certain conditions a lump-sum subsidy (fixed contract) will perform better than a production-based subsidy (variable contract). While this observation is indeed true, their work does not seek to derive a contract that is globally optimal for entire market. Our analyses will suggest that a superior contract is to offer a bunched solution where a portion of the market picks up the lump-sum subsidy while production-based subsidy is picked up by vendors with better technology. The intuition being that their constraint on non-positive taxes or non-negative subsidies is similar to our market constraint that no prices can be charged for personalization services and hence a bunched solution becomes optimal.

Several managerial implications result from our analyses. Our findings show that one advantage of employing toolbars for personalization i.e., the ability to offer a fixed, take-it or leave-it contract, is superior to a consumer-welfare maximizing variable contract *only* under narrowly defined circumstances. If a market is even marginally tilted in favor of convenience-seekers and there is some variation in consumers' privacy concerns, Web portals are better off by offering a toolbar of maximum length and allow consumers to turn-off of features based on their privacy concerns. Indeed currently, most portals including Yahoo!, Google and others pursue this strategy, where consumers are free to choose a subset of services. With regards to coupons and incentives, our results clearly point out that they should be employed to further incentivize consumers who already find personalization and sharing information to be somewhat comfortable, rather than to incentivize privacy-seekers to use these services. This is an important recommendation since many online retailers assume that consumers' switching costs alone are enough for continuous sharing of information and focus primarily on new and tentative users.

While a majority of portals are going the direction of pick-and-choose variable service levels for their toolbars, Amazon.com is are pursuing a different approach today. A couple of years back (pre-toolbar era) Amazon introduced incentives in the form of discount coupons based upon consumers' clicking through its Gold Box services and sharing certain information. This feature has since been dropped and intuitively this can be explained as an ill-conceived combination of fixed coupons with variable services, i.e., consumers were allowed to choose their level of personalization but they received the same benefits. No new usage/preference information was obtained by Amazon through this service as consumers would have anyways used these services. However, with its use of A9.com's toolbar and services, and Amazon.com uses a fixed-contract approach where its privacy policy clearly tells users that once the toolbar is downloaded a certain fixed amount of information will be monitored and subsequent usage will be personalized. More recently Amazon.com has also introduced coupons as incentives for usage of this service, e.g., an instant reward of 1.57% (called ($\pi/2$% by Amazon) of purchases after adequate usage of A9.com services. Along the lines of our discussion vendor's sharing surpluses with consumers through coupons, Amazon.com clearly specifies "*How can we afford this? –*

Sponsored links revenue — from the small text-based ads on A9.com and Amazon.com search results pages — will help offset costs we incur through the Instant Reward promotion. With our automatic π/2% Instant Reward, we are effectively sharing with you some of the money we collect from sponsored links, i.e. sharing the pi." Although, at this time Amazon.com will not reveal for what level of services usage this coupon will become effective but clearly one can observe that these are the beginning stages of constructing a combination of fixed and variable contracts.

A final managerial conclusion for personalization markets with privacy concerns is that if the firm is not offering coupons, and if the market is nascent implying a great dispersion in consumer types, it is perhaps best to allow consumers to choose their own optimal service-levels. However if couponing is a possibly, a vendor cannot go wrong by offering variable contracts so as to attract convenience-seeking types and simultaneously offer fixed-contracts for the lower-end of the market.

4.3 Future work

Perhaps an immediate follow-up to our paper is one that studies the design of incentives for personalization with competiting firms that differ in their trust-level, i.e., some manner in which firms can influence privacy costs. While not discussed in this paper, there are also many markets with price as a strategic variable, where goods can have a satiation point that might vary across consumers in the market. For example, increasing number of components bundled together may fall in utility due to a user's resource constraints from his computing capacity. Thus, our model can be extended to study a monopolist's bundling problem under usage constraints.

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Appendix

Proof of Lemma 1 (VS-NC & VS-VC): The vendor's profit function in VS-NC is given by $\Pi_{VS-NC} = \int_{\underline{R}}^{\bar{R}} (\alpha\psi S(r)) f(r) dr$, and under full information, the vendor will want to extract full surplus from each consumer, hence the IR for all types will be binding, i.e., $pS(r) - \psi^2 r S^2(r) = 0$. This gives $S^*_{VS-NC}(r) = \frac{p}{\psi^2 r}$, and $\Pi^*_{VS-NC} = \frac{\alpha p}{\psi} \int_{\underline{R}}^{\bar{R}} \frac{f(r)}{r} dr$. The vendor's profit function in VS-VC is given by $\Pi_{VS-VC} = \int_{\underline{R}}^{\bar{R}} (\alpha\psi S(r) - T(r)) f(r) dr$ and the binding IR for all types in this case implies $pS(r) - \psi^2 r S^2(r) + T(r) = 0$. Substituting $T(r) = -pS(r) + \psi^2 r S^2(r)$ in the profit function, we have

$$\Pi_{VS-VC} = \max_{S,T} \int_{\underline{R}}^{\bar{R}} \left(\alpha\psi S(r) + pS(r) - \psi^2 r S^2(r)\right) f(r) dr \tag{A1}$$

Employing point-wise maximization, we need to maximize only the terms inside the integral. Hence we have

$$\frac{\partial}{\partial S(r)}\left(\alpha\psi S(r) + pS(r) - \psi^2 r S^2(r)\right) = 0 \Rightarrow S^*_{VS-VC}(r) = \frac{\alpha\psi + p}{2\psi^2 r} \tag{A2}$$

Substituting this for the transfer, we get $T^*_{VS-VC}(r) = \frac{(\alpha\psi + p)(\alpha\psi - p)}{4\psi^2 r}$. Substituting these values in the profit function, we have

$$\Pi^*_{VS-VC} = \int_{\underline{R}}^{\bar{R}} \left(\alpha\psi \frac{\alpha\psi + p}{2\psi^2 r} - \frac{(\alpha\psi + p)(\alpha\psi - p)}{4\psi^2 r}\right) f(r) dr \Rightarrow \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_{\underline{R}}^{\bar{R}} \frac{f(r)}{r} dr \tag{A3}$$

Hence we have Lemma 1.■

Proof of Lemma 2 (fs-nc): Consumers that derive positive utility for the offered fixed service level will contract and let r_{fs-nc} be consumer who is indifferent between taking the contract and not participating, since $\frac{\partial u(r,s)}{\partial r} < 0 \,|_{\forall s>0}$, we can safely conclude that all consumers whose privacy sensitivity is *less* than $r = \frac{p}{\psi^2 s_{fs-nc}}$ will participate, i.e., the market given by $r \in [\underline{R}, r_{fs-nc}]$ will be served. Hence we maximize the profits $\pi^*_{fs-nc} = \max_s \alpha\psi \int_{\underline{R}}^{r_{fs-nc}} sf(r)\,dr$ for this region. The FOC gives:

$$\frac{\partial}{\partial s}\alpha\psi s \int_{\underline{R}}^{r_{fs-nc}} f(r)\,dr = \frac{\partial}{\partial s}\alpha\psi s F\left(\frac{p}{\psi^2 s}\right) \Rightarrow \alpha\psi F\left(\frac{p}{\psi^2 s}\right) + \alpha\psi s\left(\frac{-p}{\psi^2 s^2}\right) f\left(\frac{p}{\psi^2 s}\right) = 0$$

$$\Rightarrow \psi F\left(\frac{p}{\psi^2 s}\right) - \frac{p}{\psi s} f\left(\frac{p}{\psi^2 s}\right) = 0 \Rightarrow s^*_{fs-nc} = \frac{p}{\psi^2}\frac{f\left(\frac{p}{\psi^2 s^*_{fs-nc}}\right)}{F\left(\frac{p}{\psi^2 s^*_{fs-nc}}\right)} \Rightarrow s^*_{fs-nc} = \frac{p}{\psi^2} h\left(r^*_{fs-nc}\right)$$

Checking for the SOC, we have

$$\pi''_{fs-nc} = \frac{\partial^2}{\partial s^2}\alpha\psi s \int_{\underline{R}}^{r_{fs-nc}} f(r)\,dr \Rightarrow \frac{\partial}{\partial s}\alpha\psi\left(F\left(\frac{p}{\psi^2 s}\right) - \frac{p}{\psi^2 s} f\left(\frac{p}{\psi^2 s}\right)\right)$$

$$\alpha\psi\left(-\frac{p}{\psi^2 s^2} f\left(\frac{p}{\psi^2 s}\right) - \frac{p}{\psi^2 s}\left(-\frac{p}{\psi^2 s^2}\right) f'\left(\frac{p}{\psi^2 s}\right) + \frac{p}{\psi^2 s^2} f\left(\frac{p}{\psi^2 s}\right)\right) \Rightarrow \frac{\alpha p^2}{\psi^3 s^3} f'\left(r_{fs-nc}\right)$$

We can see that for $\pi''_{fs-nc}\left(s^*_{fs-nc}\right) < 0$ implies we need to have $\frac{\alpha p^2}{\psi^3 s^3} f'\left(r^*_{fs-nc}\right) < 0$. Note that f is a unimodal distribution in the interval $[\underline{R}, \overline{R}]$, hence $\pi''_{fs-nc}\left(s^*_{fs-nc}\right) < 0$ if $r^*_{fs-nc} >$ mode of f.

For profits $\pi^*_{fs-nc} = \alpha\psi \int_{\underline{R}}^{r^*_{fs-nc}} s^*_{fs-nc} f(r)\,dr \Rightarrow \frac{\alpha p}{\psi}\frac{f\left(\frac{p}{\psi^2 s^*_{fs-nc}}\right)}{F\left(\frac{p}{\psi^2 s^*_{fs-nc}}\right)} F\left(\frac{p}{\psi^2 s^*_{fs-nc}}\right) \Rightarrow \frac{\alpha p}{\psi} f\left(r^*_{fs-nc}\right)$. ■

Proof of Lemma 3 (fs-fc): Similar to Lemma 2, all consumers whose privacy sensitivity is less than $r_{fs-fc} = \frac{ps+t}{\psi^2 s^2}$ will participate, hence the portion of the market given by $r \in [\underline{R}, r_{fs-fc}]$ will be served. For the optimal solutions we need to consider FOC's of $\pi^*_{fs-fc} = \max_{s,t} \int_{\underline{R}}^{r_{fs-fc}} (\alpha\psi s - t) f(r)dr$ w.r.t to s and t and simultaneously solve for optimal values.

The FOC with respect to s gives: $\frac{\partial}{\partial s}\int_{\underline{R}}^{\frac{ps+t}{\psi^2 s^2}} (\alpha\psi s - t) f(r)dr \Rightarrow \frac{\partial}{\partial s}(\alpha\psi s - t) F\left(\frac{ps+t}{\psi^2 s^2}\right) = 0$

$$\alpha\psi^3 F\left(\frac{ps+t}{\psi^2 s^2}\right) - (\alpha\psi s - t)\left(\frac{p}{s^2} + \frac{2t}{s^3}\right) f\left(\frac{ps+t}{\psi^2 s^2}\right) = 0 \qquad \text{(A4)}$$

The FOC with respect to t gives: $\frac{\partial}{\partial t}\int_{\underline{R}}^{\frac{ps+t}{\psi^2 s^2}}(\alpha\psi s - t)f(r)dr \Rightarrow \frac{\partial}{\partial T}(\alpha\psi s - t)F\left(\frac{ps+t}{\psi^2 s^2}\right) = 0$

$$-F\left(\frac{ps+t}{\psi^2 s^2}\right) + \left(\frac{\alpha\psi s - t}{\psi^2 s^2}\right)f\left(\frac{ps+t}{\psi^2 s^2}\right) = 0 \qquad \text{(A5)}$$

Now solving equations (A4) and (A5) simultaneously, we have

$\frac{\alpha\psi}{s^2} - \left(\frac{p}{s^2} + \frac{2t}{s^3}\right) \Rightarrow (\alpha\psi - p)s - 2t = 0 \Rightarrow \quad t = \frac{(\alpha\psi - p)s}{2}$, i.e., $s^*_{fs-fc} = \frac{p+\alpha\psi}{2\psi^2}h\left(\frac{p+\alpha\psi}{2\psi^2 s^*_{fs-fc}}\right)$

Hence we have $s^*_{fs-fc} = \frac{p+\alpha\psi}{2\psi^2}h\left(r^*_{fs-fc}\right)$, $t^*_{fs-fc} = \frac{(\alpha\psi - p)s^*_{fs-fc}}{2}$. Substituting these values in the profit function $\pi^*_{fs-fc} = \int_{\underline{R}}^{r^*_{fs-fc}}\left(\alpha\psi s^*_{fs-fc} - t^*_{fs-fc}\right)f(r)dr$, we get $\pi^*_{fs-fc} = \left(\frac{\alpha\psi + p}{2\psi}\right)^2 f\left(r^*_{fs-fc}\right)$. ■

Proof of Proposition 1: The first part of this proposition is rather easy to see as $t^*_{fs-fc} > 0$ only when $\alpha\psi > p$ and indeed the service levels of Lemma 2 and 3 converge when $\alpha\psi = p$ (note that this true in case of Lemma 1 as well). Further $r^*_{fs-nc} = \frac{p}{\psi^2 s^*_{fs-nc}}$ and $r^*_{fs-fc} = \frac{\alpha\psi + p}{2\psi^2 s^*_{fs-fc}}$. Substituting for s^*_{fs-nc} and s^*_{fs-fc} respectively, we get $r^*_{fs-nc} = \frac{1}{h\left(r^*_{fs-nc}\right)}$ and $r^*_{fs-fc} = \frac{1}{h\left(r^*_{fs-fc}\right)}$. Since the function h is common to both cases, the 45 degree line can cut $1/h$ at only one point and hence $r^*_{fs-nc} = r^*_{fs-fc}$. We can also see that the vendor's profits are clearly higher in Lemma 3. We also know that the that $u\left(r^*_{fs-nc}\right) = U\left(r^*_{fs-fc}\right) = 0$. Let $r^*_{fs} = r^*_{fs-nc} = r^*_{fs-fc}$, the surplus for the most convenience seeking consumer $(\underline{R})$ in fs-nc case can be written as

$u_{fs-nc}(\underline{R}) = \frac{p^2}{\psi^2}h\left(r^*_{fs}\right) - \underline{R}\frac{p^2}{\psi^2}h^2\left(r^*_{fs}\right) \Rightarrow \frac{p^2}{\psi^2}h\left(r^*_{fs}\right)\left(1 - h\left(r^*_{fs}\right)\right)$. Similarly the surplus for the same consumer in the fs-fc case can be written as

$U_{fs-fc}(\underline{R}) = ps^*_{fs-fc} - \underline{R}\psi^2\left(s^*_{fs-fc}\right)^2 + \frac{(\alpha\psi - p)s^*_{fs-fc}}{2} \Rightarrow \frac{(\alpha\psi + p)}{2}s^*_{fs-fc} - \underline{R}\psi^2\left(s^*_{fs-fc}\right)^2$

$\Rightarrow \frac{(\alpha\psi + p)}{2}\frac{(\alpha\psi + p)}{2\psi^2}h\left(r^*_{fs}\right) - \underline{R}\psi^2\frac{(\alpha\psi + p)^2}{4\psi^4}h^2\left(r^*_{fs}\right) \Rightarrow \frac{(\alpha\psi + p)^2}{4\psi^2}h\left(r^*_{fs}\right)\left(1 - \underline{R}h\left(r^*_{fs}\right)\right)$. Since $\alpha\psi > p$ for couponing to be in effect, we can clearly see that $U_{fs-fc}(\underline{R}) > u_{fs-nc}(\underline{R})$. We also know that both U_{fs-fc} and u_{fs-nc} are linear in r, hence $U_{fs-fc}(r) > u_{fs-nc}(r)$ for all $r \in \left[\underline{R}, r^*_{fs}\right)$, i.e.

every consumer but the last $\left(r_{fs}^*\right)$ will get a greater surplus with couponing with the last served consumer getting zero surplus in both cases. Since the profits and consumer surplus are higher with couponing, the social welfare is also higher in this case. ■

Proofs of Lemma 4 (vs-nc) and Proposition 2: In the absence of any coupons, the only instrument that the vendor has in the variable case is the service level. Ideally, the vendor would like to offer the service level in Lemma 1, i.e, $s(r) = \frac{p}{\psi^2 r}$, where he extracts full surplus from the consumers. However if he declares this to be his service offering, all consumers of type r will misrepresent their type to be some $\tilde{r} = 2r$, such that they get their surplus maximizing service level of $\frac{p}{2\psi^2 r}$. In general, for any $s(r)$ offered by the vendor, the consumers will declare their type such that $s(\tilde{r}) = \frac{p}{2\psi^2 r}$, or $\tilde{r} = s^{-1}\left(\frac{p}{2\psi^2 r}\right)$. Hence the vendor has no option but to offer $s(r) = \frac{p}{2\psi^2 r}$. And in fact if the vendor were to consider a variable-service, fixed coupon, the consumer will lie and declare the same $\tilde{r}$ as above and will get the incentive as well. Similarly if some fixed service s_f is offered with some variable incentives $t(r)$, the consumer will always lie such that his utility $ps_f - r\psi^2 s_f^2 + t(r)$ is maximized. In fact all of them will declare an $\tilde{r} = t'^{-1}(0)$, i.e., at which $t(r)$ is maximized. ■

Proof of Lemma 5 (vs-vc): To derive the truth-revealing variable contracts, we first need to set up the consumer ICs for some services and incentive schedule $\{s(.), t(.)\}$. Let a consumer of type r declares some $\tilde{r}$, in order for the direct revelation mechanism to be truthful, we need

$$U(s(r)) \geq U(s(\tilde{r})) \Rightarrow t(r) + ps(r) - \psi^2 rs^2(r) \geq t(\tilde{r}) + p(\tilde{r}) - \psi^2 rs(\tilde{r}) \qquad \text{(A6)}$$

for any $(r, \tilde{r})$ in $[\underline{R}, \overline{R}] \text{x} [\underline{R}, \overline{R}]$. Equation (A6) implies $\forall$ pairs (r, r') in $[\underline{R}, \overline{R}] \text{x} [\underline{R}, \overline{R}]$, we need

$$t(r) + ps(r) - \psi^2 rs(r) \geq t(r') + ps(r') - \psi^2 rs(r') \qquad \text{(A7)}$$

$$t(r') + ps(r') - \psi^2 r's(r') \geq t(r) + ps(r) - \psi^2 r's(r) \qquad \text{(A8)}$$

Adding equations (A7) and (A8), we get

$$r'(s(r) - s(r')) \geq r(s(r) - s(r')) \Rightarrow (r' - r)(s(r) - s(r')) \geq 0 \qquad \text{(A9)}$$

Thus the incentive compatibility constraint requires that the schedule of services $s(.)$ has to be non-increasing, and also meets the almost everywhere differentiability conditions prescribed in Laffont and Martimort (2002) for both the service and incentive schedules. From equation (A6), we know that in order for response $\tilde{r}$ to be optimal for a consumers type r

$$\dot{t}(\tilde{r}) + p\dot{s}(\tilde{r}) - 2\psi^2 r s(\tilde{r})\dot{s}(\tilde{r}) = 0 \qquad \text{(A10)}$$

For truth to be an optimal response for type r it must be the case that

$$\dot{t}(r) + p\dot{s}(r) - 2\psi^2 r s(r)\dot{s}(r) = 0 \qquad \text{(A11)}$$

Equation (A11) must hold for all r in $\left[\underline{R}, \overline{R}\right]$ since r is unknown to the vendor.

Our optimal solution must also satisfy the local second order conditions

$$\ddot{t}(\tilde{r})|_{\tilde{r}=r} + p\ddot{s}(\tilde{r})|_{\tilde{r}=r} - 2\psi^2 r\left(\dot{s}^2(\tilde{r}) + s(\tilde{r})\ddot{s}(\tilde{r})\right)|_{\tilde{r}=r} \le 0 \qquad \text{(A12)}$$

$$\ddot{t}(r) + p\ddot{s}(r) - 2r\psi^2\left(\dot{s}^2(r) + s(r)\ddot{s}(r)\right) \le 0 \qquad \text{(A13)}$$

Differentiating equation (A11) with respect to r, we have

$$\ddot{t}(r) + \dot{s}(r) + p\ddot{s}(r) - 2r\psi^2\left(\dot{s}^2(r) + s(r)\ddot{s}(r)\right) = 0 \qquad \text{(A14)}$$

And therefore we can write equation (A13) as

$$\dot{s}(r) \le 0 \qquad \text{(A15)}$$

The crossing property or Spence-Mirrlees Condition requires that the cross-derivative has a constant sign. Since $\frac{\partial^2 u(s,p,r,\psi)}{\partial s \partial r} = \frac{\partial\left(p - 2r\psi^2 s\right)}{\partial r} = -2\psi^2 s$, which is negative for $\forall s > 0$, the condition is met. This implies that satisfying local incentive constraints will mean that global incentive constraints will also be met.

Now we will derive the total rent (consumer surplus) that the vendor will pay each consumer

$$U(r) = t(r) + ps(r) - r\psi^2 s^2(r) \qquad \text{(A16)}$$

The local incentive constraint can now be written as

$$\dot{U}(r) = \dot{t}(r) + p\dot{s}(r) - 2r\psi^2 s(r)\dot{s}(r) - \psi^2 s^2(r) \qquad \text{(A17)}$$

As $\dot{t}(r) = -p\dot{s}(r) + 2\psi^2 r s(r)\dot{s}(r)$ from the first order condition in (A11), by employing the Envelope Theorem, we have the slope of the rent as

$$\dot{U}(r) = -\psi^2 s^2(r) \qquad \text{(A18)}$$

We can now write the vendor's objective function as following

$\pi = \int_{\underline{R}}^{\overline{R}} [\alpha\psi s(r) - t(r)] f(r) dr$, subject to

$$\dot{U}(r) = -\psi^2 s^2(r) \qquad \text{(A19)}$$

$$\dot{s}(r) \leq 0 \tag{A20}$$

$$U(r) \geq 0 \tag{A21}$$

Since $\bar{R}$ is the most privacy-seeking consumer that is served by this contract, from equation (A19) the participation constraint equation (A21) simplifies to

$$U(\bar{R}) \geq 0 \tag{A22}$$

The participation constraint of the most privacy-seeking consumer will be binding $U(\bar{R}) = 0$. Momentarily ignoring equation (A20) and solving equation (A19), we get $U(\bar{R}) - U(r) = -\psi^2 \int_r^{\bar{R}} s^2(x)dx$. And since $U(\bar{R}) = 0$, we have

$$U(r) = \psi^2 \int_r^{\bar{R}} s^2(x)dx \tag{A23}$$

Using equation (A23) we can write the vendor's objective function as

$$\begin{aligned} \pi &= \int_{\underline{R}}^{\bar{R}} \left[\alpha\psi s(r) - U(r) + ps(r) - \psi^2 r s^2(r)\right] f(r)dr \\ &\Rightarrow \int_{\underline{R}}^{\bar{R}} \left[\alpha\psi s(r) + ps(r) - \psi^2 r s^2(r) - \psi^2 \int_r^{r^*_{vs-vc}} s^2(x)dx\right] f(r)dr \end{aligned} \tag{A24}$$

Using Fubini's theorem, we can rewrite the integral inside the integral as $\int_{\underline{R}}^{\bar{R}} \left(\psi^2 \int_r^{2\underline{R}} s^2(x)dx\right) f(r)dr \Rightarrow \int_{\underline{R}}^{\bar{R}} \left(\psi^2 \int_{\underline{R}}^{r} s^2(x)f(r)dr\right)dx$. Equation (A24)can be rewritten as $\Rightarrow \int_{\underline{R}}^{\bar{R}} \left(\psi^2 s^2(x)[F(r)]_{\underline{R}}^{r}\right)dx \Rightarrow \int_{\underline{R}}^{\bar{R}} \left(\psi^2 s^2(x)F(r)\right)dx$

$$\int_{\underline{R}}^{\bar{R}} \left[\alpha\psi s(r) + ps(r) - \psi^2 r s^2(r) - \psi^2 s^2(r)\frac{F(r)}{f(r)}\right] f(r)dr \tag{A25}$$

$$\int_{\underline{R}}^{\bar{R}} \left(\alpha\psi + p - \psi^2 r s(r) - \psi^2 s(r)\frac{F(r)}{f(r)}\right) s(r)f(r)dr \tag{A26}$$

Employing point-wise maximization, we need to only maximize the portion inside the integral with respect to $s(r)$. This gives

$$s^*_{vs-vc}(r) = \frac{\alpha\psi + p}{2\psi^2\left(r + \frac{F(r)}{f(r)}\right)} \Rightarrow s^*_{vs-vc}(r) = \frac{\alpha\psi + p}{2\psi^2}\frac{1}{\left(r + \frac{1}{h(r)}\right)} \tag{A27}$$

From equation (A23), we get $U(r) = \psi^2 \int_r^{\bar{R}} \frac{(\alpha\psi + p)^2 h^2(x)}{4\psi^4 (1 + xh(x))^2}dx$, simplifying we get

$$U^*_{vs-vc}(r) = \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_r^{\bar{R}} \frac{dx}{\left(\frac{1}{h(x)} + x\right)^2} \tag{A28}$$

Substituting the above in equation (A16), we get

$t(r) = -p\frac{\alpha\psi + p}{2\psi^2}\frac{h(r)}{(1 + rh(r))} + r\frac{(\alpha\psi + p)^2}{4\psi^2}\frac{h^2(r)}{(1 + rh(r))^2} + \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_r^{\bar{R}} \frac{h^2(x)}{(1 + xh(x))^2}dx$ which is

$$t^*_{vs-vc}(r) = \frac{\alpha\psi + p}{2\psi^2}\left(\frac{-p}{\left(r + \frac{1}{h(r)}\right)} + \frac{(\alpha\psi + p)}{2}\left(\frac{r}{\left(r + \frac{1}{h(r)}\right)^2} + \int_r^{\bar{R}} \frac{dx}{\left(x + \frac{1}{h(x)}\right)^2}\right)\right) \tag{A29}$$

From equation (A27) and (A29), we have Lemma 5. ■

Proof of Corollary 1: We also know that for the most privacy sensitive consumer $(\bar{R})$, $U(\bar{R}) = 0$. Since $U(\bar{R}) = u(\bar{R}) + t(\bar{R})$, if the consumer enjoys positive utility from usage of personalization services alone, i.e., if $u(\bar{R}) > 0$, we know that $t(\bar{R})$ has to be negative. Checking for $u(\bar{R})$ who receives $s(\bar{R})$, we have $ps(\bar{R}) - r\psi^2 s^2(\bar{R}) \Rightarrow s(\bar{R})(p - r\psi^2 s(\bar{R}))$. So if $t(\bar{R})$ has to be negative then it will suffice to show that $p - r\psi^2 s(\bar{R}) > 0$, i.e., $p - r\psi^2 s(\bar{R}) > 0 \Rightarrow p > \bar{R}\psi^2 \frac{\alpha\psi + p}{2\psi^2}\left(\bar{R} + \frac{1}{h(\bar{R})}\right)^{-1}$. This implies

$$p\left(1 + \frac{2}{\bar{R}h(\bar{R})}\right) > \alpha\psi \tag{A30}$$

Equation (A30) implies that if $\alpha\psi > p\left(1 + \frac{2}{\bar{R}h(\bar{R})}\right)$, then $t^*_g(\bar{R}) > 0$.

Applying a similar approach to ensuring $U(\underline{R}) > 0$, i.e. at least the least privacy sensitive consumer gets positive surplus, we need to ensure that

$$\alpha\psi > p\left(1 + \frac{2}{\underline{R}h(\underline{R})}\right) \tag{A31}$$

Since $\frac{1}{h(\underline{R})} = \frac{F(\underline{R})}{f(\underline{R})}$ and $F(\underline{R}) = 0$, we get $U(\underline{R}) > 0$ if and only if $\alpha\psi > p$. Combining equations (A30) and (A31), and since $h(\bar{R}) = f(\bar{R})$ as $F(\bar{R}) = 1$ we get Corollary 1. ■

Proof of Lemma 6: In order to derive the formal bunching solution, let the vendor offer a variable services contract $\{s_{vf}(r), t_{vf}(r)\}$ to consumers defined by $r \in [\underline{R},...,r_{vf}]$, i.e., $(\dot{s}(r) < 0, \dot{t}(r) < 0)$ and provide a bunched contract $\{s_{vf}(r_{vf}^*), t_{vf}(r_{vf}^*)\}$ to serve consumers defined by $r \in [r_{vf},...,\overline{r}_f]$, i.e., $((\dot{s}(r) = 0, \dot{t}(r) = 0))$, where r_{vf}^* is the solution to:

$$r_{vf}^* = \arg\max_{r_{vf}} \int_{\underline{R}}^{r_{vf}} (\alpha\psi s(r) - t(r)) f(r)\,dr + (\alpha\psi s(r_{vf}) - t(r_{vf})) \int_{r_{vf}}^{\overline{r}_f^*} f(r)\,dr \tag{A32}$$

Since we have to employ point-wise maximization for the first integral, we will arrive at the same menu of contracts as in Lemma 5, i.e., $\{s_{vf}^*(.), t_{vf}^*(.)\}$ will be the same as $\{s_{vs-vc}^*(.), t_{vs-vc}^*(.)\}$ albeit applied to different limits. Thus substituting the optimal menu in equation (7), we can write

$$r_{vf}^* = \arg\max_{r_{vf}} \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_{\underline{R}}^{r_{vf}} \frac{dr}{(rf(r) + F(r))} + (\alpha\psi s(r_{vf}) - t(r_{vf})) \int_{r_{vf}}^{\overline{r}_f^*} f(r)\,dr \tag{A33}$$

Subject to:

$$U(\overline{r}_f^*) = ps_{vf}^*(r_{vf}) - \overline{r}_f^* \psi^2 (s_{vf}^*(r_{vf}))^2 + t_{vf}^*(r_{vf}) = 0 \tag{A34}$$

Equation (A34), gives $\overline{r}_f^* = \dfrac{ps_{vf}^*(r_{vf}) + t_{vf}^*(r_{vf})}{\psi^2 (s_{vf}^*(r_{vf}))^2}$. Hence we can write equation (A33) as

$$\begin{aligned} r_{vf}^* = \arg\max_{r_{vf}} & \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_{\underline{R}}^{r_{vf}} \frac{dr}{(rf(r) + F(r))} \\ & + (\alpha\psi s_{vf}^*(r_{vf}) - t_{vf}^*(r_{vf})) \left(F\left(\frac{ps_{vf}^*(r_{vf}) + t_{vf}^*(r_{vf})}{\psi^2 (s_{vf}^*(r_{vf}))^2} \right) - F(r_{vf}) \right) \end{aligned} \tag{A35}$$

And the maximized profits will be given by

$$\pi_{vf}^* = \left(\frac{\alpha\psi + p}{2\psi}\right)^2 \int_{\underline{R}}^{r_{vf}^*} \frac{dr}{(rf(r) + F(r))} + (\alpha\psi s_{vf}^*(r_{vf}^*) - t_{vf}^*(r_{vf}^*)) \left(F\left(\frac{ps_{vf}^*(r_{vf}^*) + t_{vf}^*(r_{vf}^*)}{\psi^2 (s_{vf}^*(r_{vf}^*))^2} \right) - F(r_{vf}^*) \right) \blacksquare$$