Competition for Information under Privacy Concerns

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Abstract

Due to inherent privacy concerns, online personalization services such as those offered through toolbars and desktop widgets, are characterized by "no-free-disposal" (NFD) property in that more services are not necessarily better for the consumer. There are two defining characteristics of this market: First, these services are "free" as firms value consumers' preference information shared for personalization; and second, while some firms provide toolbars of a fixed-length as a take-it or leave-it offer, many others provide consumers with the option of choosing a subset of the portfolio of services offered. Our findings suggest that in a fixed-services duopoly where firms are endowed with sufficiently different marginal values for information (MVIs), the high MVI firm caters to convenience seekers in the market while the low MVI firm serves a portion of largely privacy seeking consumers in equilibrium. On the other hand, if the duopoly were characterized by sufficiently high MVIs, the firms would minimize differentiation and offer the same number of services. However, when two high-MVI firms pursue variable-services strategy, there is a unique symmetric equilibrium that maximizes consumer surplus. Counter to intuition, some very high-MVI firms may prefer the consumer-surplus maximizing strategy of offering the full set of variable services over the fixed-services strategy, thus maximizing both consumer and social welfares. Our results lead to important managerial and policy implications and interesting extensions to the existing location models.

Keywords: personalization, privacy, spatial competition, Nash-equilibrium, welfare analysis

1 Introduction

As a recent Federal Trade Commission (FTC) workshop suggests, a set of technologies that have begun to raise consumer concerns of privacy are browser-embedded toolbars, while some groups characterized this as spyware, many others tout toolbar-enabled personalization as the future of online services (FTC). A toolbar is a Browser Helper Object (BHO), wherein once downloaded and embedded it has the ability to monitor and report usage information (including Web sites visited, information filled in online forms, etc.) as well as to tailor future online interactions. More recently, Google's Deskbar, Microsoft's Live (now bundled with Vista) and Yahoo!'s Konfabulator technologies have begun to offer these services right from the desktop and are expected to replace current Web-based static personalization. Once embedded, firms can disallow consumers from turning off select features, thus consumers may need to remove the entire toolbar (and hence forgo all personalization) if their comfort level in sharing information is below that of the amount acquired by the toolbar. As the FTC continues to assess its position on these technologies, these technologies have become ubiquitous with every online portal (Yahoo!, MSN, AOL, etc.) and many large online firms (eBay, Amazon, etc.) offering them, e.g. currently over 618 Million searches originate from consumers using toolbars. Thus, one important objective of this paper is to investigate the regulatory implications regarding the allowance of such personalization technologies through a social welfare analysis.

The operational basis for these online firms that rely on consumer information is unique. Personalization services are entirely free of charge to the consumers; portal-like firms rely on their ability to sell browsing profiles to advertisers and targeted marketers (Dewan et al., January 05-08) while e-tailers use information acquired for personalization to manage their own inventory, marketing goals, and to enhance customer satisfaction and loyalty (Shankar et al.). Indeed, the FTC also acknowledges the legitimate use of consumer information by businesses provided that such usage can also be beneficial to consumers (See workshop report (FTC)). From the consumer behavior perspective, online personalization is an example of goods with "no free disposal" (NFD) property, an economic classification of goods meaning that more of the good is not necessarily better. Recent marketing research on pricing access services online (Essegaier et al., Spring), observes, "*Unlike physical goods for which "free disposal" is always an option and more is, in general, always better, service delivery is intrinsically participatory. Participation requires time commitment and physical effort on the part of consumers. Thus, there is no free disposal for service, and time cost and physical efforts limit the effectiveness of price incentives in altering consumer usage habit*." An important cost intrinsically related to the usage of personalization services is the privacy costs that individuals incur when sharing their preference and usage information needed for tailoring services to their tastes (Volokh, August). Therefore even if free of cost, not all consumers will prefer all services offered by the firm. This property poses many unique challenges to online firms and portals who incur costs of creating personalized services so as to acquire consumers' usage information.

Thus, our paper examines a competitive market for privacy where firms have to strategically choose their level of personalization services offering for acquiring consumer information.

Further, we incorporate the possibility of online firms being similarly or differently endowed in their capacity to generate revenue or lower operating costs from mining and using consumer information and thus they may vary in the value placed on consumer information. For example, many large firms such as AOL and MSN do not simply resell their information, but they operate their own advertising networks with their own profiling technologies as well. On the other hand, many smaller firms and portals mostly act as a carrier of other's advertising networks such as from DoubleClick (DART network – at this time Google intends to purchase DoubleClick) and Atlas (Atlas Suite). Offering personalization is not costless and these technologies are ubiquitous and available to all, hence firms need to investigate their optimal service offerings when their competitors can also potentially offer identical services. Firm strategies in this regard have largely been unexplored in academic research, particularly given the NFD nature of consumers' utility from personalization amidst their privacy concerns. In order to study the welfare implications of allowing toolbar/deskbar-like technologies, we examine the market equilibrium when one or both firms offer variable services wherein a consumer is allowed the option to choose the full set of services in the toolbar or disable some services based on their privacy concerns. We contrast this with equilibrium findings when firms offer a fixed set of services wherein consumers are made a take-it or leave-it offer to embed a toolbar of fixed-length or to not use it all. The latter scenario is of particular relevance to privacy-groups that are concerned about the possibility of consumers sharing more information than they would ideally want.

1.1 Review of relevant literature

A recent work in Management Science calls for the study of online personalization particularly in the presence of privacy concerns (Murthi and Sarkar, October). Some have posited that a market for preference information will emerge where consumers will engage in transactions that involve privacy concerns (Rust et al.). In this regard, behavioral work has suggested that consumers are more likely to accept loss of privacy if it accompanies some benefit (Laufer and Wolfe), and consumer's decision to share information is based on some cost-benefit tradeoffs, known as the "privacy calculus" (Culnan and Bies). Recent analytical work has abstracted this privacy calculus in the context of personalization through consumers' personalization for privacy (p4p) tradeoff (Chellappa and Shivendu, Chellappa and Shivendu). However, to the best of our knowledge, there has been no academic research that examines non-price competition in a market for goods with NFD property, such as in the markets for personalization services.

Murthi and Sarkar (October) point out the need to explore the segmentation aspects to a personalization market, and observe that when competing firms provide free products, differentiation along the lines of price competition becomes irrelevant. This paper addresses these aspects in that we model a market where firms differentiate themselves on the level of personalization services offered even when no price is being charged. Further, we model a market that exhibits vertical segmentation characteristics in two ways: First, producing higher level of services is increasingly costly (convex cost of producing a good along the lines of Mussa and Rosen , and many others). Second, we consider a distribution of consumers' effective marginal value for services (captured through their p4p tradeoff), similar to cases where consumers' marginal valuation for quality is distributed in a vertically segmented market.

However, note that for most goods in a vertically segmented market more quality (or any valued attribute) is strictly preferred by all consumers, i.e. while the marginal value for quality may differ amongst consumers, more is always better. Therefore, consumer utilities are typically strictly increasing in quality or *monotonic* (non-decreasing) concave. On the contrary, an important aspect of the personalization market is the NFD property of the services, where the general principle being that more is not always preferred to less. In such a market, every consumer has an ideal level of services he would like to use, similar to the ideal location point in a spatial market. This ideal point is a result of internalizing the tradeoff between gain from personalization and loss from privacy, which leads to a *non-monotonic* utility function that first increases then falls in services consumed (purely quality models with monotonic utilities are sometimes called "infinite ideal-point" – consumers homogeneously prefer an infinite amount of the good – models (Moorthy, August)). While somewhat less studied than their "free disposal" counterpart, we commonly come across NFD goods when users have an intrinsic disutility from consuming beyond their satiation level (Nahata et al.). Classic examples include identifying the right package size of travelers' toiletries (larger sizes are not always optimal), ideal level of sweetness of a drink (over-sweetness may not be preferred) and the optimal units of power production for a township (production creates pollution) (Rothwell and Rust). Hence our model of competition also exhibits properties of horizontally segmented markets.

Findings in purely vertically segmented markets are well known in economics, marketing and information systems (which consider segmentation with zero marginal cost of serving additional consumers). Commonly, segmentation by producing both high and low quality products is superior to only producing one product type except under shutdown conditions wherein a market comprises of largely high-type consumers. Given the non-price nature of competition and NFD property of the good, these findings need to be re-examined in the market for information. On the other hand, findings in horizontally segmented markets, commonly described through spatial competition in Hotelling's linear market or Salop's circular market, are diverse and highly dependent on specific assumptions on transport costs and reservation prices. We shall discuss extensions to the linear market as it is most relevant to our model setup while recognizing that in all the models described below, firms *do not* incur any production cost or any other cost of locating themselves anywhere in the market. The many extensions to Hotelling's model focus on the existence (or lack thereof) of equilibrium in a duopoly. For example, D'Aspremont et al. (Sept., 1979) show that the original Hotelling model with linear transport costs does not result in an equilibrium solution of minimum differentiation; rather convex transport costs lead to a price equilibrium in any location pair and firms will attempt to *maximize* their differentiation by locating as far away from each other as possible. While our model does have the equivalent of convex transport costs, D'Aspremont's assumptions of infinite reservation prices (resulting in a fully covered market) do not apply to our case. In this regard, Economides' and Moorthy's setup of a market with finite reservation prices is closer in principle to ours. In particular, Economides' observation that the real source of non-existence of equilibrium in such markets is the non-quasiconcavity (caused by infinite reservation prices), not merely the discontinuity of the payoff functions, is most relevant. As we shall discuss in our results, some degree of monopolistic competition observed by his work is possible under narrowly defined circumstance in our market as well.

There are, however, also a limited number of papers that consider some aspects of *both* vertical and location-based competition. For example, Gabszewicz and Thisse (Mar., 1986) consider a market where there are vertical and horizontal differentiations with quadratic transport costs albeit with infinite reservation prices for consumers. While they exogenously consider an "inside and outside-location" (firms placed inside vs. outside consumers' distribution on a street) game, in our model firms endogenously determine their location with respect to the consumer distribution. However, one of their results where an equilibrium condition is also socially optimal rings true for a particular case of competition in our model as well and is discussed at length in the paper. A more recent paper by Desai in marketing has also considered a market characterized by both vertical and horizontal segmentation, where consumers are distributed along two attributes (marginal value for quality and taste preferences) that are independent of each other. This is perhaps the model setup closest to ours in spirit, but note that our model characterization is not a result of distributing two different consumer characteristics; rather, it is the distribution of p4p ratio combined with the NFD property of the good that brings out the vertical and horizontal aspects in our model. The above differences combined with the non-price, zero marginal cost and zero versioning cost of services in our market results in findings that are very dissimilar to Desai's model of price-competition for durable goods. While the durable goods model finds that high valuation segments are more likely to get their preferred attribute level, our results suggest the opposite – either the entire market is satisfied at its preferred level or under other circumstances the low types are more likely to be satisfied.

Interestingly, Moorthy's work on product and price competition has intermediate steps that appear structurally very similar to ours even if the motivation, model formulation and results are very different. However, note that even if there are no spatial characteristics to Moorthy's market (no ideal locations), there are structural similarities in the payoff and best response functions between his model and one of the cases in our model. For reasons discussed later, both models have two discontinuities in the payoff function that distinctly depend on whether one firm's quality (services in our model) is the same, equal to or more than the other firm's quality offering. This combined with the fact that the payoffs are concave in one's own quality (services) and linearly decreasing in the other's quality (services) creates the impression of similarity in the model. In summary, our model is distinct from the ones discussed above in that:

1. We model a good with NFD property that intrinsically generates a consumer utility function that is non-monotonic concave in service usage. Previous works generally model durable goods with free disposal or some strictly preferred property.
2. We model a market with non-price competition, thus restricting the strategic instrument available to the firms. While not identical, this allows us to draw parallels with a purely location game or the location subgame within a price-location game.
3. In our model, firms incur a quadratic fixed cost of offering services, i.e., incur a cost of locating themselves. While purely spatial models have no firm costs associated with locating, vertically segmented markets serve as a comparison point.

4. There are no marginal costs of serving additional consumers *and* there are no costs associated with offering any service level that is a subset of the maximum service produced by the firm. This property is unique to online services and brings out the costless versioning property explored elsewhere (Varian).
5. No assumption is made *a priori* on whether a market is covered or not. Along the lines of a couple of works discussed earlier, consumers have finite reservations. Similar to Moorthy , firm characteristics relative to distribution of consumer parameters is derived to determine whether a market will be covered or not in equilibrium. However, no assumptions are made on relative firm characteristics while earlier works have generally modeled identical firms in the duopoly.

In §2 we present the basic model setup when firms pursue fixed vs. variable services and briefly discuss the monopoly results. In §3, we derive equilibrium service offerings for different duopolistic markets where one or both firms pursue fixed (or variable) services strategy. We conclude this section with an investigation of welfare implications for each type of the market setup discussed earlier. §4 examines the policy implications of our findings and concludes with a discussion of theoretical and managerial implications of our analyses.

2 The model

Our model develops the strategic interactions between online firms and consumers of personalization services. Consumers engage in a privacy calculus in their decision to use personalization services as they incur privacy costs in sharing information needed for this activity (Culnan and Bies). This willingness to share information is based on the consumer's perceived benefits of disclosure balanced with its risks (Derlega et al.). Consumer behavior in this context has been modeled by prior research (Chellappa and Shivendu) as a function of consumers' marginal value for personalization p and their coefficient of information privacy concerns r given by $u(i,s,p,r) = ps - ri^2$, where s is the level of personalization services consumed and i is the preference information that needs to be shared by the consumer in order for services to be personalized.

The number of personalized services that can be created from a unit of information is commonly a function of the prevalent personalization and data mining technologies (Raghu et al., Winer). One can view this as a production function wherein some technology determines how many services (s) can be created from some information (i) that is provided. While theoretically more than one service can be created for every unit of information shared, it is generally accepted that this technology is still evolving despite significant advances in information acquisition (Chen and Hitt, September 2002). Hence we assume that one unit of preference information shared provides one unit of corresponding personalized service $(i = s)$. Since the firms determine the number of personalization services to be offered in the market, and through the usage of services the consumers determine how much information they will share, we can write a consumer c's utility as a function of personalization services consumed

$$u_c(s, p, r) = p_c s - r_c s^2 \tag{1}$$

It is important to note two salient aspects of consumption here: First, services are provided for free. Consumers do not pay any price to the firm. Second, more services are not necessarily better. Each consumer has an optimal service level $\left(s_c^*\right)$ that she prefers over all other service levels. The latter (and its quadratic form) is a characteristic of a class of economic goods with the "no free disposal" property which implies that individuals derive disutility from consuming additional quantities beyond their satiation level (Nahata, Kokovin and Zhelobodko).

Thus the utility function is non-monotonic (an inverted-U function) in services consumed, and is characterized by the two levels, the utility maximizing $s_c^* = \arg\max_s u_c(s, p, r)$ and break-even $\left(s_c^0 : u(p, r, s)\big|_{s=s_c^0} = 0\right)$ service levels respectively. For the utility function described by equation (1), we can see that $s_c^0 = \frac{p_c}{r_c}$ and $s_c^* = \frac{p_c}{2r_c}$. The ratio $\frac{p}{r}$ is known as the consumers' personalization for privacy (p4p) ratio and is a critical parameter for analysis of consumer behavior as it determines both their indifference and optimal service levels. Empirical research finds that consumers may vary in their value for personalization and concerns for privacy (Chellappa and Sin, April). Without loss of generality, we consider a market where consumers are uniformly distributed in their p4p ratio given by $\frac{p}{r} \sim U[0, b]$. This also allows us to represent the two consumer-behavior characterizing levels along the same dimensions given by $s_c^0 \sim U[0, b]$ and $s_c^* \sim U\left[0, \frac{b}{2}\right]$. We shall generally refer to consumers with low p4p ratios as privacy-seekers and those with high p4p values as convenience-seekers. A summary explanation of major notations used in this paper is presented in Table 1 in the appendix.

2.1 Online firm strategies

Firms vary in their ability to use consumer information by virtue of the extent to which this information can be exploited to their own purposes, represented by their marginal value for information (MVI). For example, portals such as Yahoo! and AOL that run their own advertising networks do not simply resell usage/preference information; rather they have a portfolio of advertising related products unlike portals such DogPile and AskJeeves. Yahoo! Search Marketing division offers products such as Search Optimizer and Marketing Console that are geared towards small to medium firms, and provides a fully customized advertising program for firms that have a budget of over $10,000 per month. Similarly, retailers like Amazon.com that carry many product categories and engage in cross-selling strategies have greater MVI's than firms that sell one type of product or use their service to host advertisements.

Offering personalization services is not costless; either firms incur their own costs of building a toolbar, or they incur licensing and technology costs from buying from firms such as BestToolBars.net and ezToolbar.com. In addition, firms offering personalization also incur costs of licensing content, building trust through alliances with trusted third-parties (e.g., TRUSTe, WebCPA, Verisign), and implementing security mechanisms to comply with FTC requirements (FTC 2000) and special legislative requirements such as Children's Online Privacy Protection Act (COPPA) and Health Insurance Portability and Accountability Act (HIPPA) as well (Anonymous, Jun 2001, Bloom et al., Scott). Hence we construct a firm's profit as

$$\pi_j = \sigma_j A(s) - s^2 \tag{2}$$

where σ_j is the marginal value for information (MVI) of a firm j and $A(s)$ is the aggregate information acquired from the usage of s personalization services. In this paper we assume that firms incur similar costs in offering services but are heterogeneous in their marginal values for information. The identical cost function not only rules out a trivial explanation that any difference in firm strategies is due to differences in costs, but is also consistent with the ubiquitous availability and open-standard nature of personalization technologies. The ability to use information however is indeed a function of firms' business strategies and endowments and may affect their overall personalization offerings.

The fixed and variable personalization service offerings of online firms can be operationalized in one of the following two ways through their toolbar (or desktop) technologies:

1. A toolbar of fixed length – where the firm contracts to personalize a fixed number of services and will monitor and acquire information corresponding to the entire set of services. The consumer is faced with a take-it or leave-it offer where they will use as long as the utility is non-negative for the service level, i.e., $s \in \left(0, s_c^0\right]$. A9.com's (affiliated with Amazon.com) toolbar is a classic example of this approach where a consumer has to agree to all the information being monitored (which is fully disclosed in the firm's privacy policy) or to not use the toolbar at all. Through out the paper we shall refer to this as the fixed-services strategy.
2. Toolbars of variable lengths – where the firm offers a toolbar with its full list of services and allows consumers to choose a subset. In this case when consumers vary in their p4p tradeoffs, each consumer will use a different services level according to her optimal level s_c^* if available, else the level provided by the firm (formally $\min\left\{s_c^*, s\right\}$). Many toolbars including Google and Yahoo! follow this approach where consumers have the option to turn off personalization based on increasingly sensitive information, e.g., the PageRank (called Web Rank in Yahoo!) feature can be removed when using Google toolbar services. We shall refer to this as the variable-services strategy.

In a monopoly, it might appear that the firm would prefer a fixed-services approach to a variable one as the latter involves letting consumers choose their preferred level while in the former the firm is assured that all consumers who participate will use the level prescribed by the toolbar. However, we have to note that in the fixed-services approach a portion of the market may not be served if the service level is too high for some consumers (more than their break-even levels), and the surplus extracted from the participating consumer is reduced if the service is too low. This tradeoff combined with the cost of offering services will clearly point towards an optimal service offering for this strategy, given by maximizing $\pi_m^F = \sigma\left(\int_s^b sU\left(s_c^0\right)ds_c^0\right) - s^2$. However, the strategy to offer variable-services can be more complex due to both the NFD nature of personalization services contrasted with the zero-marginal costs of both serving additional consumers and versioning. While the NFD property implies that consumers may use fewer services than those prescribed by the full toolbar, the firm also gets a greater number of people to participate – in fact all consumers will participate under variable services. Thus the firm will evaluate the increased market size, differential surplus acquired from each consumer, and the cost of offering the full set of services in determining its optimal service level, given by maximizing $\pi_m^V = \sigma\left(\int_o^s s_c^* U\left(s_c^*\right) ds_c^* + \int_s^{\frac{b}{2}} sU\left(s_c^*\right) ds_c^*\right) - s^2$.

Solving for optimal services by maximizing the respective profit functions, we find the optimal services and profits to be the same for the monopolist in both cases $\left(\left\{s_m^{V*}, \pi_m^{V*}\right\} = \left\{s_m^{F*}, \pi_m^{F*}\right\} = \left\{\frac{\sigma b}{2(\sigma + b)}, \frac{b\sigma^2}{4(b + \sigma)}\right\}\right)$, although the size of the market served and consumer welfare are different. While in the variable-services strategy the entire market $\frac{p}{r} \in [0, b]$ is served, under fixed offering only the market given $\frac{p}{r} \in \left[\frac{\sigma b}{2(\sigma + b)}, b\right]$ will be covered, i.e., a monopolist might be indifferent between the two strategies as the marginal loss in surplus extracted from each consumer is compensated against the gain in market size when variable services approach is used. However, firms' strategies and equilibrium outcome in a competitive market may be drastically different; not only will the portion of market served by one firm depend on the relative levels of services offered by *both*, but the equilibrium will also depend on whether one or both firms engage in the same or different strategies (fixed or variable).

3 Competition in a duopoly

We consider a duopoly where the two firms have MVIs given by σ_1 and σ_2, no assumption is made on the relative values of the two MVIs at this juncture. We consider a game in which both firms simultaneously choose their respective service levels $s_1 \in S_1$ and $s_2 \in S_2$. Note that the strategy spaces are also bounded by b $(S_1, S_2 = [0, b])$ as no consumer would use beyond this level and hence no firm will ever consider a strategy of offering services beyond this limit.

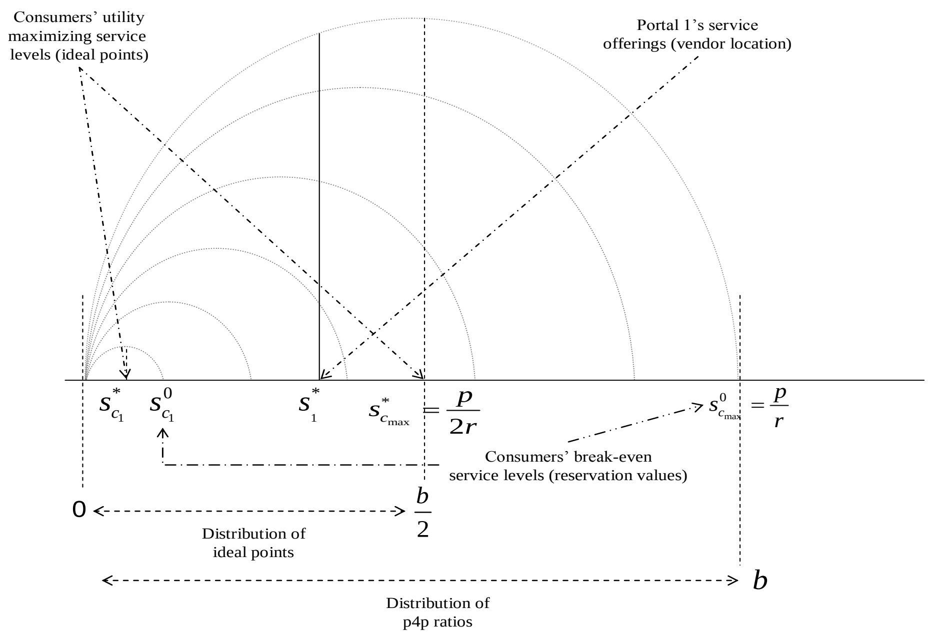

Figure 1: Distribution of consumer preferences and firm locations on a line

Figure 1 characterizes our personalization market as a linear one where each consumer's location or ideal service level s_c^* is uniformly distributed from 0 to $\frac{b}{2}$. If a firm offers a certain service level s_1 at some distance x from the ideal point of a consumer, we can see that the disutility given by $|\, u_c\left(s_c^*\right) - u_c\left(s_1\right) |$ will be rx^2. In other words, consumers suffer a convex transportation cost along the lines of D'Aspremont et al. (Sept., 1979) for which equilibrium in locations exists under certain condition. While firms incur convex costs of locating themselves on the line (normally ignored in purely spatial models), the zero-marginal costs and zero versioning costs of services (a lower service-level can be offered costlessly once a toolbar of higher services is built) combined with the NFD property create unique competitive situations non-existent in physical goods markets. While the fixed services approach appears to be structurally similar to physical goods model setup (i.e., once a firm has located, all consumers have to buy from that point), the variable-services strategy creates a unique possibility wherein if the firm offers a service level s_1, he can costlessly serve all consumers with $s_c^* < s_1$ at their ideal levels. Again, we do not assume *a priori* as to whether the market is covered (mostly the case in spatial models) or not, i.e., consumers do not have infinite reservation. In view of these differences, it is not clear if any equilibrium possibilities exist at all, hence this is an interesting model to analyze from a location model perspective as well.

3.1 Market outcomes when <u>both</u> firms offer fixed services

We first consider the case when both firms offer a toolbar of fixed length, i.e., a take-it or leave-it offer where consumers who pick up the contract agree to the acquisition of a fixed amount information on their usage. In this case, consumers will use the level of service that is provided so long as their utilities are non-negative. In the duopoly, a consumer will choose Firm 1 if her utility from using s_1 is greater than that from using s_2 $(u_c(s_1) > u_c(s_2))$. First consider the case when Firm 1 might offer fewer services than Firm 2 ($s_1 < s_2$, case "a"). A consumer will derive a higher utility from using services provided by Firm 1 if:

$$ps_1 - rs_1^2 > ps_2 - rs_2^2 \Rightarrow \frac{p}{r}(s_1 - s_2) > s_1^2 - s_2^2 \qquad (3)$$

And since $s_1 < s_2$, equation (3) implies $\frac{p}{r} < s_1 + s_2$. Notice that consumers with $s_c^0 < s_1$ would not use any services at all, therefore consumers whose break-even service level $s_c^0 \in [s_1, s_1 + s_2)$ would use Firm 1's services and the remaining consumers $s_c^0 \in [s_1 + s_2, b]$ would use Firm 2's services. By symmetry, we know that if Firm 1 offers more services than Firm 2 ($s_1 > s_2$, case "c"), consumers with $s_c^0 \in [s_1 + s_2, b]$ will use Firm 1's services. If both firms offer the same level of service level ($s_1 = s_2$, case "b"), then given that consumers are indifferent between the two firms, Firm 1 will get half the market of all consumers using the services, i.e. half of the consumers whose break-even service level are $s_c^0 \in [s_1, b]$. Therefore, the amount of information that a firm acquires depends upon both his level of service and its magnitude relative to that of the second firm. We can formally write Firm 1's profit functions[1] as

$$\pi_1^F = \begin{cases} \pi_{1a}^F = \sigma_1 \int_{s_1}^{s_1+s_2} s_1 U\left(s_c^0\right) d\left(s_c^0\right) - {s_1}^2 & \text{if } \ (s_1 < s_2) \\ \pi_{1b}^F = \frac{1}{2}\sigma_1 \int_{s_1}^{b} s_1 U\left(s_c^0\right) d\left(s_c^0\right) - {s_1}^2 & \text{if } \ (s_1 = s_2) \\ \pi_{1c}^F = \sigma_1 \int_{s_1+s_2}^{b} s_1 U\left(s_c^0\right) d\left(s_c^0\right) - {s_1}^2 & \text{if } \ (s_1 > s_2) \end{cases}$$

[1] Alphabets in the subscripts of the profit functions correspond to the respective cases regarding the relative service levels of the firms as discussed above.

By symmetry, we can construct Firm 2's profit function and notice that the payoff functions of both firms are discontinuous in the service space. The discontinuity could lead one to believe that there may be no equilibrium in pure strategies at all and that only mixed strategies equilibria exist. However, for our analyses, we consider only pure strategy equilibria for two reasons: First, mixed strategies severely limit the explanatory power of the model; second, work by Dasgupta and Maskin (Jan., 1986) suggests that it is not the discontinuity itself, but rather failure of the payoff functions to be quasi-concave that is the reason for the non-existence of equilibrium in pure-strategies. They propose that under certain conditions (quasi-concavity, upper semi-continuity and graph continuity of the payoff functions), even a game with functions that have limited continuity can possess a pure-strategy Nash equilibrium. Later work has argued these conditions are far too restrictive and only certain conditions[2] on the aggregator function need to be satisfied for a pure strategy equilibrium to exist (Baye et al.). In the appendix we show that our profit functions satisfy these conditions.

Thus, we go on to develop the strategic interactions between the two firms so as to find pure-strategy equilibria. Firm 1's strategy is a best response to the strategy of Firm 2 if it maximizes $\pi_1^F\left(\max\left\{\pi_{1a}^F, \pi_{1b}^F, \pi_{1c}^F\right\}, s_2\right)$ in the strategy space S_1 for any given s_2. In considering the best response of Firm 1, not only does he need to decide on the service level but he also needs to determine whether to offer a service level that is lower than, equal to or greater than the competing firm. By symmetry, we can see that Firm 2 also needs to make a similar decision in responding to services offered by Firm 1. Independently, profits in their defined regions are all strictly concave, hence interior optima are candidates for equilibrium outcomes. However, note that for some firm parameters the functions do not attain their maximum within the defined regions, e.g., when $\sigma_1 > 2b$, π_{1a}^F is still increasing as s_1 approaches s_2, implying that this firm type will attempt to "undercut" Firm 2 by offering slightly more services. Hence when a firm's MVI is greater than $2b$, offering services fewer than those offered by his competitor can never be a profit-maximizing strategy in equilibrium. Similarly, when a competitor offers a service level $s_2 \geq \dfrac{b\sigma_1}{2b + 3\sigma_1}$, irrespective of his own MVI, it is not optimal for Firm 1 to offer more than Firm 2 as π_{1c}^F is monotonically decreasing in s_1; hence region c cannot be an

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[2] The sufficient conditions are *Diagonal Transfer Continuity* and *Diagonal Transfer Quasiconcavity.* See Appendix for details.

equilibrium candidate for this particular firm pair. Therefore, if the Nash equilibrium pair is given by $s_1^{F^*}, s_2^{F^*}$, then from Firm 1's perspective and for any Firm 2 there *might be*

1. An asymmetric equilibrium where Firm 1 offers *fewer* services than Firm 2, given by $s_1^{F^*}, s_2^{F^*}\left(s_1^{F^*} < s_2^{F^*}\right) = \left\{\frac{\sigma_1}{2b} s_2^{F^*}, s_2^{F^*}\right\}$ and Firm 1's MVI is $\sigma_1 < 2b$.
2. An asymmetric equilibrium where he offers *more* services than Firm 2, given by $s_1^{F^*}, s_2^{F^*}\left(s_1^{F^*} > s_2^{F^*}\right) = \left\{\frac{\left(b - s_2^{F^*}\right)\sigma_1}{2\left(b + \sigma_1\right)}, s_2^{F^*}\right\}$ and Firm 1's MVI $\sigma_1 < \frac{2b s_2^{F^*}}{b - 3 s_2^{F^*}}$.
3. A symmetric equilibrium where he offers the same services as Firm 2, given by $s_1^{F^*}, s_2^{F^*}\left(s_1^{F^*} = s_2^{F^*}\right) = \left\{s_2^{F^*}, s_2^{F^*}\right\}$ for any MVI of Firm 1.

In order for the service-pair $\left\{s_1^{F^*}, s_2^{F^*}\right\}$ to be an equilibrium candidate, we need $\max\left\{\pi_1^F\left(., s_2^*\right)\right\} = \pi_{1a}^F\left(s_1^{F^*}\right)$ and $\max\left\{\pi_2^F\left(s_1^*, .\right)\right\} = \pi_{2c}^F\left(s_2^{F^*}\right)$ in case 1, $\max\left\{\pi_1^F\left(., s_2^*\right)\right\} = \pi_{1c}^F\left(s_1^{F^*}\right)$ and $\max\left\{\pi_2^F\left(s_1^*, .\right)\right\} = \pi_{2a}^F\left(s_2^{F^*}\right)$ in case 2, or $\max\left\{\pi_1^F\left(., s_2^*\right)\right\} = \pi_{1b}^F\left(s_1^{F^*}\right)$ and $\max\left\{\pi_2^F\left(s_1^*, .\right)\right\} = \pi_{2b}^F\left(s_2^{F^*}\right)$ in case 3. Combining 1 and 2 and by symmetry, we find that an asymmetric equilibrium can exist between two firms (suppose that Firm 1 offers the lower service level) only if $\sigma_1 < 2b$ and $\sigma_2 < \frac{2b s_1^{F^*}}{b - 3 s_1^{F^*}}$. From Figure 2, we can see that for such instances the payoffs of both firms are well behaved in that they are *continuous but for upward jumps* as defined by Milgrom and Roberts ; once again pointing towards the existence of pure-strategy Nash equilibria. In order to identify these MVI combinations for which there exists an equilibrium, we find bounds on the firm parameters (MVIs σ_1 and σ_2) that satisfy the above requirements, i.e. for which two types of firms will the market result in an equilibrium outcome. While algebraically tedious (hence the proof is relegated to the appendix), our approach provides lucid solutions to firm and regulator problems, and allows us to derive managerially relevant insights on firm competition under privacy.

Lemma 1: *When both firms offer only a fixed-service toolbar, there exists an asymmetric equilibrium given by* $\left(s_1^{F^*}, s_2^{F^*}\right) = \left(\frac{b\sigma_1\sigma_2}{4b\left(b + \sigma_2\right) + \sigma_1\sigma_2}, \frac{2b^2\sigma_2}{4b\left(b + \sigma_2\right) + \sigma_1\sigma_2}\right)$ *when the competition is characterized by one firm with low MVI* $\sigma_1 < \frac{2b}{1 + \sqrt{2}}$ *and other with a relatively higher MVI* $\sigma_2 \geq \frac{8b^2\sigma_1}{4b^2 - 4b\sigma_1 - \sigma_1^2}$.

Lemma 1 tells us that if firms are sufficiently differentiated by their marginal values for information and if one firm has a low MVI, then the two firms will share the marketplace in such a way that the low MVI firm caters to consumers with low p4p ratios and the high MVI firm caters to those with high p4p ratios. Note that when both firms offer fixed services, the NFD nature of the good does not come into play, i.e., it does not matter what the ideal points are, consumers will select a service-level as along as their individual rationality (IR) constraints are satisfied, and their choice of firm will depend on the individual's incentive compatibility (IC) constraint. We also know that for a given service level, consumer utilities are increasing in the p4p ratio, i.e., $u_1(s) > u_2(s)$ if $\left(\frac{p}{r}\right)_1 > \left(\frac{p}{r}\right)_2$. Hence along the lines of strictly vertically segmented markets, we have an equilibrium where one firm serves the low types and the other serves the high types. (Moorthy). The condition on the separation of MVI's essentially ensures that the firm with low MVI will not attempt to undercut the higher MVI competitor due to the trade off between his costs and marginal value for information.

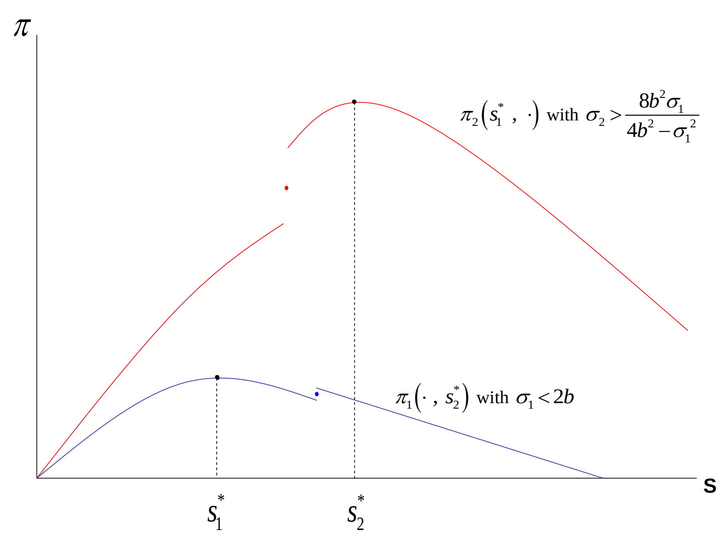

Figure 2: Profits under asymmetric equilibrium when both firms offer fixed services

Interestingly, note that while the services offered by both firms are increasing in σ_2 of the high MVI firm, the services offered by high MVI firm (s_2) is decreasing in σ_1 while the low MVI's services (s_1) continue to increase in his own MVI. The intuition behind this is that if the MVIs are sufficiently far apart, the firms will make themselves attractive to very distinct segments and as the lower MVI approaches $2b$, this firm will begin to offer services that are now attractive to some consumers (who were using more than their optimal levels) of the high MVI firm. Hence for the large MVI firm, the cost of offering high number of services is not offset by the demand captured and will therefore lower his service level. Further, we know that the number of consumers who are not served $\left(s_c^0 < s_1^{F*}\right)$ increases in MVI, while on the other hand some consumers (with high p4p ratio) might receive services closer to their optima. This portends interesting consumer (and hence social) welfare implications that we shall explore later. In fact, since $\lim_{\sigma_1 \to 2b} s_1^{F*} = s_2^{F*}$, we not only know that the threshold is important in maintaining the asymmetric equilibrium but also that there is potentially a symmetric equilibrium if the MVIs of both firms are sufficiently high.

Lemma 2: *When both firms offer only a fixed-services toolbar, then there exists a symmetric equilibrium given by* $s_F^* = \left(s_1^{F*}, s_2^{F*}\right) = \left(\frac{b}{3}, \frac{b}{3}\right)$ *when competition is characterized by both firms having high MVIs* $(\{\sigma_1, \sigma_2\} \geq 2b)$*. For all other firm characterizations (e.g., both firms with low MVI* $(\{\sigma_1, \sigma_2\} < 2b)$*, there exists* ***no symmetric equilibrium*** *even for identical MVIs.*

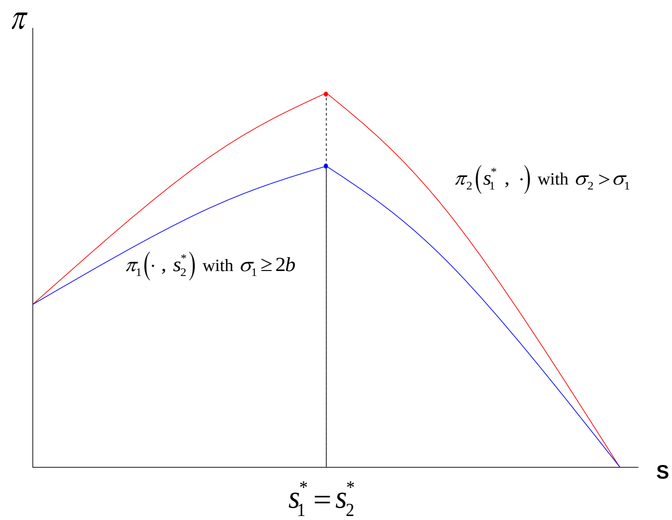

Figure 3: Profits of firms under symmetric equilibrium when both firms offer fixed services

Lemma 2 suggests that when both firms have high MVI and both offer a fixed-services take-it or leave-it offer, the only feasible equilibrium is characterized by firms offering the same level of services and sharing the market equally (see Figure 3 for a graphical representation of profits of the high MVI firms with respect to services offered). Note that not only is the equilibrium service level purely a function of consumers' p4p distribution, but also firms need not have identical MVIs for symmetric equilibrium to exist; it is only required that both firms have MVIs weakly higher than a threshold $(2b)$. This suggests that even if two firms had the ability to offer a greater number of services, doing so would make a firm attractive to consumers to the right

$\left(\text{consumers with } s_c^* \in \left[s, \frac{b}{2}\right]\right)$ while those on the left $\left(\text{with } s_c^* \in \left[\frac{b}{3}, s\right)\right)$ might begin to prefer the competitor. Essentially, above the equilibrium service level, gain from high p4p consumers' usage of services does not outweigh loss from offering services above this level. This implies that with a fixed-services approach, a portion of the market will always be left un-covered. In particular, a third of the market will not be served as consumers with break-even services below the equilibrium service level $\left(s_c^0 < \frac{b}{3}\right)$ will not use any personalization services.

The implication of this result is that even if the marginal cost of serving an additional consumer is zero, the competitive dynamics of a fixed-services approach will lead firms to maximize profits by not serving the segment of consumers with minimal value for personalization and/or high privacy concerns. At the same time some consumers $\left(\frac{b}{3} < s_c^* \leq \frac{b}{2}\right)$ are left wanting for more services as the equilibrium service level will not fully satisfy this segment. An important reason as to why firms need to possess sufficiently high MVIs for the symmetric equilibrium to exist is that when one firm is below the threshold, there is always tendency for the firms to serve different portions of the market as in Lemma 1. On the other hand, when both are below the threshold, symmetric equilibrium is not feasible either because sharing the market is never an optimal strategy. The simple intuition is that since consumers are indifferent between the services offered by the two firms as long as they offer the same level of services, both firms incur the full infrastructure costs while only getting half the market and firms could always increase this market size by offering slightly more or fewer number of services. Another important implication is one that hints at reducing consumer privacy concerns. We can see that profits of both firms are increasing in consumers' p4p ratios and prior research (Chellappa and Sin, April) suggests that engendering trust in a personalization context may reduce privacy concerns. While it is beyond the scope of this paper, one could observe that even if service offerings are indistinguishable, firms may better their profits by differentiating themselves on the basis of consumer trust.

3.2 Market outcomes when both firms offer variable services

We now consider the more common scenario where firms offer a toolbar of certain length, but allow consumers to use only a subset of services by turning off information acquisition for some services thus forgoing personalization benefit from these services as well. The NFD property of the good plays an important role here in that with the option of choosing their own service levels, consumers will choose only their optimal service level

s_c^* if available. Since consumers will be indifferent between the services offered by the two firms, both firms will share the consumer segment with $s_c^* \le \min\{s_1, s_2\}$. The remaining consumers $\left(s_c^* > \min\{s_1, s_2\}\right)$ will use services from the firm offering a higher service level because they can no longer be satisfied by the other firm. However, note that this segment of consumers can only use their ideal level of services up to the level offered by the firm with more services; beyond which they can only use the exact amount that is offered. Thus, we can formally write Firm 1's profit functions as

$$\pi_1^V = \begin{cases} \pi_{1a}^V = \dfrac{1}{2}\sigma_1 \displaystyle\int_0^{s_1} s_c^* U\left(s_c^*\right) d\left(s_c^*\right) - s_1^{\,2} & \text{if } \left(s_1 < s_2\right) \\ \pi_{1b}^V = \dfrac{1}{2}\sigma_1 \left[\displaystyle\int_0^{s_1} s_c^* U\left(s_c^*\right) d\left(s_c^*\right) + s_1 \int_{s_1}^{\frac{b}{2}} U\left(s_c^*\right) d\left(s_c^*\right)\right] - s_1^{\,2} & \text{if } \left(s_1 = s_2\right) \\ \pi_{1c}^V = \dfrac{1}{2}\sigma_1 \displaystyle\int_0^{s_2} s_c^* U\left(s_c^*\right) d\left(s_c^*\right) + \sigma_1 \left[\int_{s_2}^{s_1} s_c^* U\left(s_c^*\right) d\left(s_c^*\right) + s_1 \int_{s_1}^{\frac{b}{2}} U\left(s_c^*\right) d\left(s_c^*\right)\right] - s_1^{\,2} & \text{if } \left(s_1 > s_2\right) \end{cases}$$

We can observe that for some firm parameters, offering a service level lower than that of the competitor is a strictly *dominated* strategy; when $\sigma_1 \le 2b$, π_{1a}^V is negative regardless of the service level offered by Firm 2. The intuitive reason is that when both firms offer variable services, the firm offering lower service level incurs the full cost of offering the service while being assured of only half the market corresponding to that service level. On the other hand, if $\sigma_1 > 2b$, π_{1a}^V is still increasing as s_1 approaches s_2, implying that this firm will prefer to offer the same or higher number of services as his competitor. Extending this logic to Firm 2 and by symmetry we can easily preclude the possibility of an asymmetric equilibrium when variable services define the market. Therefore if the Nash equilibrium pair is given by s_1^{V*}, s_2^{V*}, then from Firm 1's perspective and for any Firm 2 there *might* be

1. A symmetric equilibrium where he offers the same services as Firm 2, given by $s_1^{V*}, s_2^{V*}\left(s_1^{V*} = s_2^{V*}\right) = \left\{s_2^{V*}, s_2^{V*}\right\}$ for any MVI of Firm 1
2. No asymmetric equilibrium.

In order for the service-pair $\left\{s_1^{V*}, s_2^{V*}\right\}$ to be an equilibrium candidate, we need to have $\max\left\{\pi_1^V\left(., s_2^*\right)\right\} = \pi_{1b}^V\left(s_1^{V*}\right)$ and $\max\left\{\pi_2^V\left(s_1^*, .\right)\right\} = \pi_{2b}^V\left(s_2^{V*}\right)$. Similar to the previous cases, we derive boundaries on the firm parameters (σ_1, σ_2) so as to explore for the possibility for any equilibrium strategy.

Lemma 3: *When both firms allow consumers to self-select their respective service levels, only a* <u>*symmetric equilibrium*</u> *is possible; which exists only when the competition is characterized by two high MVI firms* $(\{\sigma_1, \sigma_2\} > 2b)$. *The equilibrium number of services offered by each firm will be the full set, given by* $s_V^* = \left(s_1^{V*}, s_2^{V*}\right) = \left(\frac{b}{2}, \frac{b}{2}\right)$. *For all other firm types there is no market equilibrium even if their MVIs are identical.*

This situation is unique to our model that is characterized by NFD property of the services, and the zero marginal and versioning costs of offering variable services. The intuition behind Lemma 3 is as follows: since consumers are indifferent between the two firms and are free to choose their individual desired levels of personalized services, if two firms offer different service levels, then the firm offering the higher service level would get half of market of its competitor and will further capture the entire segment of consumers whose personalization needs are higher than the competitor's offering $\left(\text{i.e. consumers with } s_c^* \in [\min\{s_1, s_2\}, \max\{s_1, s_2\}]\right)$. While offering more services than the competitor appears preferable, because no consumer has s_c^* greater than $\frac{b}{2}$, no firm would offer services higher than this level. Hence, if the MVIs are high enough for firms to offset the cost of offering such a high level of personalization, both firms would offer the maximum level of services desired by the consumers in equilibrium. Thus the equilibrium service level is independent of the firms' own MVIs (as long as they are above the threshold) and all consumers enjoy their ideal level of personalization services.

It is salient to note the difference between the full lengths of the toolbar (the maximum number of personalization services offered) in the two symmetric equilibria described in Lemmas 2 and 3, given by $\frac{b}{3}$ and $\frac{b}{2}$ respectively. While the market is not covered under the fixed-services case, not only are all consumers being served but consumer welfare is also *maximized* in the latter case. While a monopolist would be indifferent between offering a fixed toolbar strategy or one that allows consumers to choose their preferred level, in a competitive model it is not clear yet whether firms with high MVIs will necessarily prefer one over the other. Intuitively, it might appear that firms would prefer to set the service levels and consumers use the prescribed level, consistent with price setting behavior in most markets. However, in our context of zero marginal and versioning costs, the results might yet be surprising as even if variable services is consumer-welfare maximizing, the market is fully covered thus holding out the distinct possibility of being better for the firm than the fixed-services strategy when a third of the market is always left un-served. Before we discuss the social welfare implications of fixed and variable strategies, we shall first consider a hybrid case where one firm chooses to offer a variable-services contract while the other opts for a fixed-services one.

3.3 Market outcome when one firm pursues fixed while the other pursues variable

Without loss of generality, assume Firm 2 to be the one that offers a toolbar of fixed length, while Firm 1 allows consumers to choose services in a variable fashion. First consider the case when both firms offer different levels of services $(s_1 \neq s_2)$. If Firm 1 offers fewer number of services than 2 $(s_1 < s_2)$, then all consumers with surplus maximizing number of services lower than that offered by Firm 1 $\left(s_c^* \leq s_1\right)$ would choose Firm 1 because they can freely choose their ideal level to consume. The remaining consumers would choose Firm 1 if their utility from using s_1 is greater than that from s_2, i.e. $u_c(s_1) > u_c(s_2)$. We can see that consumers whose p4p ratio $\frac{p}{r} \in [2s_1, s_1 + s_2)$ will still use Firm 1's services. However, if Firm 1 offers more services than 2 $(s_1 > s_2)$, all consumers will choose Firm 1 and use their individual utility-maximizing number of services. If both firms offer the same level of services $(s_1 = s_2)$, Firm 1 would capture all consumers whose $s_c^* \leq s_1$ and half the market of all remaining consumers. Thus the profit function of the firm offering variable services can be written as

$$\pi_1^{\widetilde{V}} = \begin{cases} \pi_{1a}^{\widetilde{V}} = \sigma_1\left[\int_0^{s_1} s_c^* U\left(s_c^*\right) d\left(s_c^*\right) + \int_{2s_1}^{s_1+s_2} s_1 U\left(s_c^0\right) d\left(s_c^0\right)\right] - {s_1}^2 & \text{if } (s_1 < s_2) \\ \pi_{1b}^{\widetilde{V}} = \sigma_1\left[\int_0^{s_1} s_c^* U\left(s_c^*\right) d\left(s_c^*\right) + \frac{1}{2}\int_{s_1}^{\frac{b}{2}} s_1 U\left(s_c^*\right) d\left(s_c^*\right)\right] - {s_1}^2 & \text{if } (s_1 = s_2) \\ \pi_{1c}^{\widetilde{V}} = \sigma_1\left[\int_0^{s_1} s_c^* U\left(s_c^*\right) d\left(s_c^*\right) + \int_{s_1}^{\frac{b}{2}} s_1 U\left(s_c^*\right) d\left(s_c^*\right)\right] - {s_1}^2 & \text{if } (s_1 > s_2) \end{cases}$$

And the profit function of the firm offering fixed services can be written as

$$\pi_2^{\widetilde{F}} = \begin{cases} \pi_{2a}^{\widetilde{F}} = \sigma_2\left[\int_{s_1+s_2}^{b} s_2 U\left(s_c^0\right) d\left(s_c^0\right)\right] - s_2{}^2 & \text{if } \left(s_1 < s_2\right) \\ \pi_{2b}^{\widetilde{F}} = \sigma_2\left[\frac{1}{2}\int_{s_2}^{\frac{b}{2}} s_2 U\left(s_c^*\right) d\left(s_c^*\right)\right] - s_2{}^2 & \text{if } \left(s_1 = s_2\right) \\ \pi_{2c}^{\widetilde{F}} = -s_2{}^2 & \text{if } \left(s_1 > s_2\right) \end{cases}$$

Let the Nash equilibrium pair be given by $s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}$. We can immediately observe that when the firm offering higher number of services allows consumers to choose their preferred level, it is never optimal for the firm offering a smaller length toolbar to pursue a fixed-services strategy. The intuitive reason is that consumers would undoubtedly use their ideal service level if given the choice, i.e., $s_2^{\widetilde{F}*} < s_1^{\widetilde{V}*}$ is never an equilibrium possibility. Notice that similar to section 3.1, for some firm parameters the profit functions do not attain their maximum within the defined regions: when $\sigma_1 > 2b$, $\pi_{1a}^{\widetilde{V}}$ of the firm offering variable services is still increasing as s_1 approaches s_2, implying that when a firm's MVI is greater than $2b$, offering services fewer than that offered by his competitor is an unlikely equilibrium strategy. Since any asymmetric equilibrium implies $s_2^{\widetilde{F}*} > s_1^{\widetilde{V}*}$, from Firm 1's perspective and for any Firm 2 there *might* be

1. An asymmetric equilibrium where Firm 1 offers *fewer* services than Firm 2, given by $s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\left(s_1^{\widetilde{V}*} < s_2^{\widetilde{F}*}\right) = \left\{\frac{\sigma_1}{2b} s_2^{\widetilde{F}*}, s_2^{\widetilde{F}*}\right\}$ and Firm 1's MVI is $\sigma_1 < 2b$.
2. A symmetric equilibrium where he offers the same services as Firm 2, given by $s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\left(s_1^{\widetilde{V}*} = s_2^{\widetilde{F}*}\right) = \left\{s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right\}$ for any MVI of Firm 1.

In order for the service pair $\left\{s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right\}$ to be an equilibrium candidate, we need $\max\left\{\pi_1^{\widetilde{V}}\left(., s_2^*\right)\right\} = \pi_{1a}^{\widetilde{V}}\left(s_1^{\widetilde{V}*}\right)$ and $\max\left\{\pi_2^{\widetilde{F}}\left(s_1^*, .\right)\right\} = \pi_{2c}^{\widetilde{F}}\left(s_2^{\widetilde{F}*}\right)$ in case 1, or $\max\left\{\pi_1^{\widetilde{V}}\left(., s_2^*\right)\right\} = \pi_{1b}^{\widetilde{V}}\left(s_1^{\widetilde{V}*}\right)$ and $\max\left\{\pi_2^{\widetilde{F}}\left(s_1^*, .\right)\right\} = \pi_{2b}^{\widetilde{F}}\left(s_2^{\widetilde{F}*}\right)$ in case 2. Since it is never optimal for the firm pursuing fixed-services to offer fewer services than the firm pursuing variable-services strategy, the only possible asymmetric equilibrium will be characterized by $s_1^{\widetilde{V}*} < s_2^{\widetilde{F}*}$ when $\sigma_1 < 2b$.

Lemma 4: *In a duopoly where one firm pursues a fixed-services strategy while the other does not, there is no symmetric equilibrium. An asymmetric equilibrium exists if the MVI of the firm allowing variable services is low*

$\left(\sigma_1 < \frac{2b}{1+\sqrt{2}}\right)$ *and that of the one pursuing fixed-services strategy is sufficiently higher* $\left(\sigma_2 \geq \frac{8b^2\sigma_1}{4b^2 - 4b\sigma_1 - \sigma_1{}^2}\right)$. *The equilibrium pair is given by* $\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right) = \left(\frac{b\sigma_1\sigma_2}{4b(b+\sigma_2)+\sigma_1\sigma_2}, \frac{2b^2\sigma_2}{4b(b+\sigma_2)+\sigma_1\sigma_2}\right)$.

It is quite apparent that it cannot be optimal for a firm to engage in a fixed-services strategy and offer the same or fewer number of services than its competitor who allows consumers to choose their preferred levels. Also note that the service offerings are the same as in Lemma 1 (asymmetric equilibrium when both firms offer fixed services), although the difference of MVIs between the two firms needs to be larger. This differentiation is driven by both bounds, i.e., the small MVI firm is smaller than his counterpart in Lemma 1 and the large MVI firm has to be larger at the same time. Intuitively, we can see that the small MVI firm, by offering variable services, is essentially serving those consumers who would have been left un-served when both firms offered fixed services (Lemma 1), i.e., those with very low p4p ratios $\left(s_c^0 < \frac{b\sigma_1\sigma_2}{4b(b+\sigma_2)+\sigma_1\sigma_2}\right)$. Further, unlike in Lemma 1, the market will be fully covered in this situation as the worst any consumer in this market can do is to use $s_1^{\widetilde{V}*}$.

An important finding of our analyses is that in equilibrium, whenever at least one firm offers variable services, the market will always be fully covered and the consumer surplus will always be higher. The simple intuition behind this is that whenever one firm allows consumers to choose their desired service levels, every consumer can find a service level that correspond to a non-zero utility; while low types will pick up some service level lower than that offered by the variable-services firm, the high types will choose between this firm and the competitor who offers a higher service level. Thus from a consumer surplus point of view as well even if one firm offers variable-services, consumers are always better off. Further, note that when the MVI of the fixed-services firm increases, *both* firms would find it optimal to increase their service levels. While the increase in the fixed-service level may or may not lead to increased consumer surplus, the increase in the smaller MVI firm's offering will result in a greater portion of consumers in the market being satisfied at their preferred level. The equilibrium number of services as well as profits for the firms and the equilibrium regions are summarized in Table 2 (in the appendix) and Figure 4, respectively.

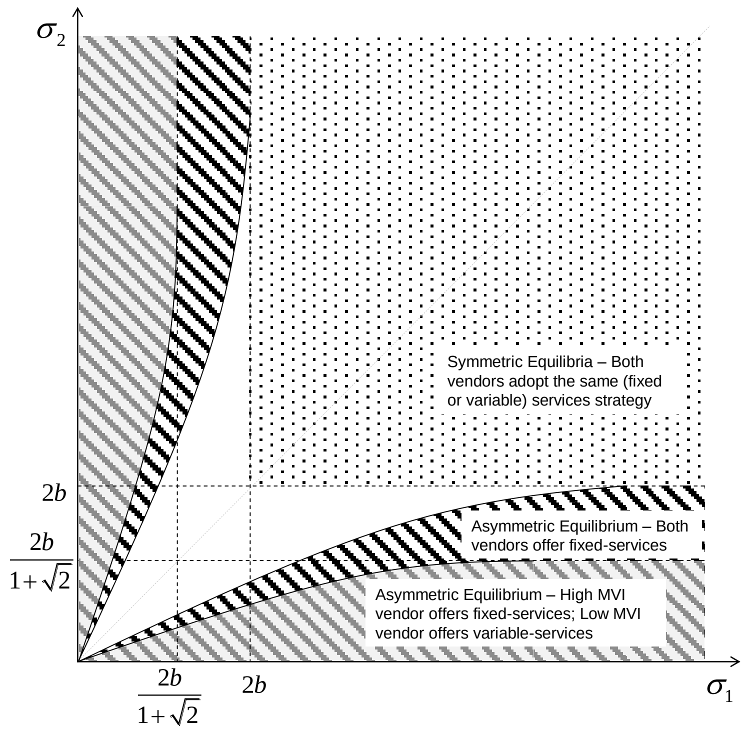

Figure 4: Equilibrium regions in the duopoly

3.4 Welfare analyses

Having derived the equilibrium solutions under different combinations of services-strategy, we shall now analyze their implications to consumer and social welfare. From our earlier discussions, it is quite evident that when one (both) firms pursue a variable services strategy, some (all) consumers enjoy their surplus maximizing level of services.

Lemma 5: *In the duopoly where there is at least one low MVI firm* $(< 2b)$ *and the higher MVI firm offers fixed-services, the market is fully covered and consumer-welfare is higher when the low MVI firm offers variable services than when both offer fixed-services. In the market characterized by two firms with sufficiently high MVIs* $(\sigma_1, \sigma_2 > 2b)$, *we observe the following:*

i. *Consumer surplus is always maximized when both firms adopt a variable-services strategy in equilibrium.*
ii. *Firm's equilibrium profits from adopting variable-services strategy is higher than their corresponding profits from engaging in fixed-services, when their respective MVIs are very high* $\sigma_1, \sigma_2 > 10b$ *and the social-welfare is the highest in this case.*
iii. *Even if firm MVIs are lower than the threshold in (ii), equilibrium social welfare in variable-services strategy is higher than that under fixed-services strategy under a lowered limit* $(\sigma_1 + \sigma_2 > 20b - 6)$

Table 3 (in the appendix) provides the consumer surplus and social welfare under each of the market conditions discussed in the paper. Parts of Lemma 5 are quite intuitive in that due to the NFD property of the good, when variable services are being offered, consumers will pick up their respective optimal service levels if available and thus raising consumer surplus. However, an interesting observation of Lemma 5 is that while variable-services are obviously good for the consumers, in some cases, they are also superior to fixed-services offerings for the firms as well (see Figure 5). The economic intuition behind this finding is that for high MVI firms, the combined loss from not serving a third of the market (consumers with low p4p ratio $s_c^0 < \frac{b}{3}$) and not catering to some convenience-seekers at their preferred level (those with $\frac{b}{3} < s_c^* \leq \frac{b}{2}$) is higher than the costs of increasing their service level from $\frac{b}{3}$ to $\frac{b}{2}$. Contributing to this observation is the fact that firms suffer *no marginal or versioning* costs in catering to consumers who choose a service level lower than the full offering. Closer to the threshold of $2b$, fixed-services is better than variable-services strategy though both profits are increasing in the firms' MVI. However, profits for firms in the variable case are increasing at a faster rate with MVI as compared to the rate of increase in the fixed case $\left(\frac{\partial \pi^{V*}}{\partial \sigma} > \frac{\partial \pi^{F*}}{\partial \sigma}\right)$. This aspect combined with the fact that consumer surplus is higher in the variable case ensures that the social welfare with variable services is higher than that of the fixed-services even before firms themselves find it optimal to offer variable-services (hence the $\sigma_1 + \sigma_2 > 20b - 6$ condition rather than $\sigma_1 + \sigma_2 > 20b$).

In this paper, we have analyzed equilibrium strategies under different cases of services-strategy adoption. From Figure 5, we can see that there is a small region describing competitive markets where no pure-strategy equilibrium exists. While this generally suggests that firms might continue to undercut each other by offering services that are perhaps not optimal, there is also a distinct possibility of mixed-strategy equilibria in this space. This would essentially suggest that firms of these types will randomize their services strategies. One main purpose of this work is to illustrate the problem from a regulator's point of view as to what the consumer and social welfare implications might be under different cases. However, one could also view our analyses as the subgame equilibrium results of a two-stage game where firms first simultaneously choose their strategy of offering (fixed vs. variable-services), and in a second stage choose their service levels. In this case our equilibrium results would be the candidates for a subgame perfect Nash equilibrium (SPNE) of the two-stage game. In order to study the equilibrium outcome of such a game, we would be comparing the profits identified in each of the Lemmas.

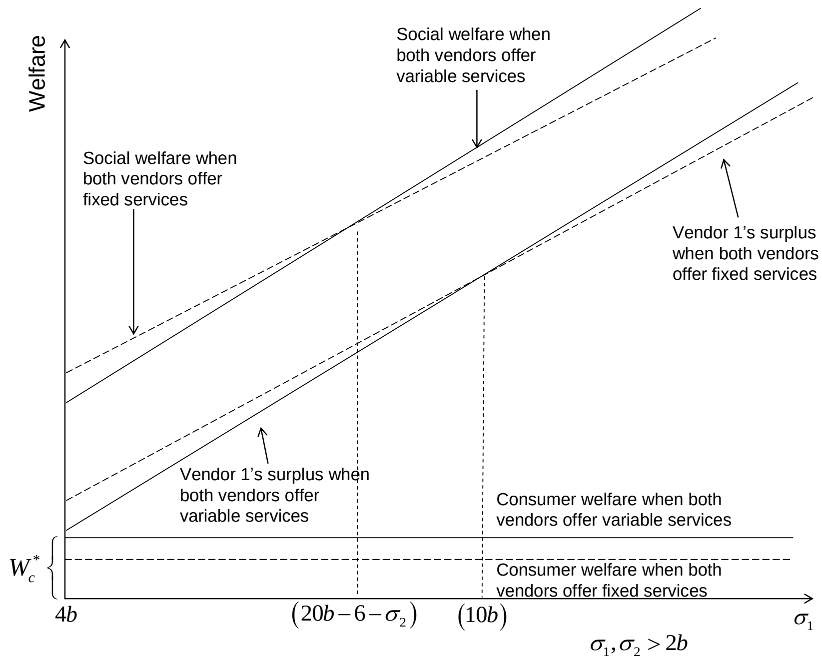

Figure 5: Welfare comparison: both firms offering fixed versus variable services

In fact, we can see that in general when a firm faces a competitor whose profits are increasing in services, then offering variable services weakly dominates offering fixed services; because offering fixed services when a competitor might offer variable will result in the firm accruing negative profits. Similarly, we could extend the model to a 3-stage game, where in the first stage the firms strategically determine their marginal values for information (MVIs) by investing in developing their own advertising networks and incurring some costs. Clearly the results of such a game would depend on the relative costs incurred by the two firms and is beyond the scope of this paper.

4 Policy implications and discussion of results

An important motivation for our problem is the emergence of new browser-embedded technologies such as toolbars that provide firms with greater control over how information about consumers' online usage is acquired. The fixed-services strategy – the option to acquire a fixed-amount of information and deliver a given set of personalized services – was hitherto non-existent as early forms of personalization was largely restricted to user controlled Web-based static mechanisms. For example, personalization through a Web interface relies largely on server logs and cookies (Murthi and Sarkar, October) and it is difficult for a firm to require consumers to use all services offered. Due to the static nature of the Web's Hyper Text Transfer Protocol (HTTP), the most control that firms enjoyed was that they could "expire" accounts that did not use enough services or share adequate information. It is in fact the fixed-services contract that a user agrees to, often with little or no control subsequently, that has gotten the attention of the media and privacy groups that compare these toolbars to spyware technologies, e.g., the recent campaign against Ask Jeeves (Stone). It is indeed this potentially detrimental level of control over how much users share and what they get the main subject of ongoing FTC investigations into the legitimate and illegitimate uses of toolbar-like technologies (FTC).

While many spurious firms are employing toolbars giving rise to their "Spyware" reputation, due to its non-intrusive nature, numerous legitimate online firms are employing it for personalizing services to their users. Currently, large firms like Yahoo! and MSN offer all of their personalization services (more than those available through their Web-based technologies) through browser-embedded toolbars. Their strategy so far has been to allow consumers to select a subset of their toolbar services. For example, Google provides a toolbar wherein one could personalize search, mail, and a variety of other services including a feature called PageRank that is considered to be highly intrusive by many. However, Google provides users with the option of using only a subset of its toolbar-based personalization, allowing users to turn-off the PageRank option and thus not collecting the associated information. This is in contrast to strategies of those in the retail space such as Amazon with its Alexa and A9.com toolbars, where Amazon does not allow consumers to choose a subset of services to personalize. Once a user downloads and embeds the A9.com toolbar, a variety of information including the Web-sites visited, products searched for, email addresses used in fill-in forms, etc., is automatically acquired and the user is then provided with a personalized list of sites and products during next usage; although currently the toolbar is focused only on products carried by its parent company, Amazon.com. The user has no control in that he cannot specify that the A9 toolbar should collect and report information (and hence personalize) only on certain services but not others.

4.1 Policy implications

Our results categorically suggest that the non-price nature of online personalization market, combined with the NFD property of services, creates a situation where the availability of variable services is always superior from a consumer surplus perspective. This is in contrast to pricing strategies for goods with free disposal, where a monopolist typically can extract more consumer surplus by offering variable rather than fixed contracts (Sundararajan, December). In our model, a monopolist is indifferent between the two strategies but in a competitive situation many equilibrium possibilities exist; each with its own consumer and social welfare implications. First of all, we note that in the case of both firms having high MVIs, a symmetric equilibrium where firms offer variable services is superior not only from the consumer-surplus perspective but also for the producer surplus and hence the overall social welfare of the market. It is evident that the adoption of fixed-strategies in equilibrium is largely a function of whether one or both firms have smaller marginal value for information.

An important policy implication of these findings is that perhaps the regulator need not outright ban the use of technologies such as toolbars, but rather ensure the participatory aspect of the consumer-firm interaction to be in favor of the consumers. Interestingly, our results suggest that the regulator needs to be more concerned with the many small firms who survive on the limited re-sale value for information than with large firms with sophisticated uses for consumer data. There is also a clear indication towards encouraging both competition and the allowance of firms to mine preference/usage information as long as consumers are made fully aware of their privacy implications. Thus an important recommendation of our work will be for a regulator like the FTC to not engage in the legislation of what technologies can or cannot be used, but rather focus on the explication of the impacts of these technologies through education and more importantly, monitor and enforce any agreements between firms and consumers.

One other suggestion might be to allow for ongoing consolidations in the firm space where many firms are beginning to merge, thus reducing the number of smaller independent firms to a few large ones. For example, Yahoo! now owns Inktomi, Overture, and Altavista; Lycos has acquired or merged with Tripod, Angelfire, Matchmaker and Wired; InfoSpace now includes Dogpile, WebCrawler, MetaCrawler and WebFetch. This is perhaps one of those unique markets where an oligopoly of a few large players might be beneficial to the consumers. Through variable-services offerings large firms also end up serving those consumers in the fringe of the market (i.e., highly privacy-sensitive consumers).

4.2 Summary

From a modeling point of view, our research adds to the literature on competition in NFD goods and services markets; and from a theoretical perspective, the competition is characterized by discontinuous payoffs that generally suggest existence of equilibrium only in mixed-strategies (Dasgupta and Maskin, Jan., 1986).

However, we find that our payoffs can also be classified as *continuous but for upward jumps* (Milgrom and Roberts), and the aggregator function (sum of payoffs across the strategy space) is both Diagonally Transfer Continuous and Diagonally Transfer Quasi-concave (Baye, Tian and Zhou). This allows us to explore more meaningful pure-strategy equilibria.

We discuss two firm strategies, one where they offer a fixed set of services to consumers and in the other where variable services are offered. When fixed-services are offered and consumers are distributed in their effective marginal value for services, the market is characterized by competition reminiscent of vertically segmented markets. Our results show the existence of a segmented market along the lines of Moorthy , where a firm endowed with a lower marginal value for information serves the low types while the higher MVI firm serves the high types. However, unlike segmented markets for goods with free disposal, we also observe a symmetric equilibrium where both firms offer the same level of services and share the market equally. The market is not covered in either case.

The NFD aspect of the good combined with the zero-marginal and zero-versioning costs have a pronounced effect when the market is characterized by variable-services offering. Unlike in the fixed-services case where non-zero utility was the main participation criteria, in the variable-services case consumers select services based on how far they are from their ideal-points (the surplus-maximizing level) and many consumers may be satisfied at their desired levels. The closest physical world examples can be found in models of franchise competition where fast-food franchises and car dealerships can locate themselves at multiple points on a linear market (Hadfield, Winter, Iyer). Our results show that such a market is only characterized by symmetric, consumer-surplus maximizing equilibria in a duopoly of high MVI firms. From a modeling perspective, this draws comparisons with outside location game similar to that in Gabszewicz and Thisse where firms place themselves on the edge of a linear market and maximize welfare.

From a managerial perspective, since firms' profits are increasing in consumers' p4p ratio, our results suggest that firms should employ significant trust building and other reassuring services that are known to help allay privacy concerns and therefore increase the p4p ratios. Since firms with large MVIs will have a strong incentive to move towards variable-services offering, it is evident that smaller independent firms that solely depend on external agencies like DoubleClick for generating value from information will find it hard to continue sustaining in this market. Perhaps these smaller firms will distinguish themselves by going the niche services route or will be absorbed into some larger firms. It is also interesting to note that while in the highly competitive marketplace firms offer variable toolbar-sizes, Amazon is currently persisting with fixed-services approach. This could perhaps be attributed to its near monopoly status in the area of retail personalization, although our model would suggest that with increasing number of firms occupying this space, Amazon will eventually allow consumers the option to choose a subset of its A9.com toolbar services.

4.3 Limitations and future research

As with any first model of a real-world context, we are limited in the number of issues we can explore in the current paper. One first such limitation is perhaps the assumption that firms only vary in their MVI and have common cost coefficients. It is quite possible that firms may also differ in their ability to personalize and also in the liability costs of the information they acquire, process and store. Further, we have assumed a simple personalization technology here where a unit of personalized service can be generated for a given unit of personal information; with rapid advances in technology perhaps more services can be produced for a single unit of information. Indeed, it would be interesting to explore how firms can differentiate themselves on the basis of the amount of services they can offer for a single piece of information rather than only the total number of indistinguishable services. Another interesting extension to our model would be the possibility of offering incentives along with information acquisition services and develop services-incentive menus for NFD goods and services.

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Competition for Customer Information: An Economic Model of Online Personalization in the Presence of Privacy Concerns

Appendix

SYMBOL	DEFINITION	SYMBOL	DEFINITION
p	Consumer's marginal value for personalization services	r	Consumer's privacy cost coefficient
σ_i	Marginal value for information (MVI) of firm *i*	$\pi_i(\sigma_i, s_i)$	Firm *i*'s profit function with respect to the MVI and level of services offered
s	Personalization services (s_c^* - consumer's surplus-maximizing service level; s_c^0 - consumer's break-even service level)	*F*	Superscript denoting fixed-services
$u(p, r, s)$	Consumers' utility from personalization services	*V*	Superscript denoting variable-services

Table 1: Key Notation

Services Strategy (Equilibrium)	Equilibrium Services	Equilibrium Profits	Condition
Both firms offer fixed services (Asymmetric Equilibrium)	$s_1^{F*} = \frac{b\sigma_1\sigma_2}{4b(b+\sigma_2)+\sigma_1\sigma_2}$ $s_2^{F*} = \frac{2b^2\sigma_2}{4b(b+\sigma_2)+\sigma_1\sigma_2}$	$\pi_1^{F*} = \frac{b^2\sigma_1^2\sigma_2^2}{[4b(b+\sigma_2)+\sigma_1\sigma_2]^2}$ $\pi_2^{F*} = \frac{4b^3\sigma_2^2(b+\sigma_2)}{[4b(b+\sigma_2)+\sigma_1\sigma_2]^2}$	$\sigma_1 < 2b$ $\sigma_2 \geq \frac{8b^2\sigma_1}{4b^2 - \sigma_1{}^2}$
Both firms offer fixed services (Symmetric Equilibrium)	$s_1^{F*} = s_2^{F*} = \frac{b}{3}$	$\pi_1^{F*} = \frac{b(\sigma_1 - b)}{9}$ $\pi_2^{F*} = \frac{b(\sigma_2 - b)}{9}$	$\sigma_1, \sigma_2 \geq 2b$
Both firms offer variable services (Symmetric Equilibrium)	$s_1^{V*} = s_2^{V*} = \frac{b}{2}$	$\pi_1^{V*} = \frac{b(\sigma_1 - 2b)}{8}$ $\pi_2^{V*} = \frac{b(\sigma_2 - 2b)}{8}$	$\sigma_1, \sigma_2 > 2b$
Firm 1 offers variable services & Firm 2 offers fixed services (Asymmetric Equilibrium)	$s_1^{\tilde{V}*} = \frac{b\sigma_1\sigma_2}{4b(b+\sigma_2)+\sigma_1\sigma_2}$ $s_2^{\tilde{F}*} = \frac{2b^2\sigma_2}{4b(b+\sigma_2)+\sigma_1\sigma_2}$	$\pi_1^{\tilde{V}*} = \frac{b^2\sigma_1^2\sigma_2^2}{[4b(b+\sigma_2)+\sigma_1\sigma_2]^2}$ $\pi_2^{\tilde{F}*} = \frac{4b^3\sigma_2^2(b+\sigma_2)}{[4b(b+\sigma_2)+\sigma_1\sigma_2]^2}$	$\sigma_1 < \frac{2b}{1+\sqrt{2}}$, $\sigma_2 \geq \frac{8b^2\sigma_1}{4b^2 - 4b\sigma_1 - \sigma_1{}^2}$

Table 2: Equilibrium outcomes under different services strategies

Type of Competition	Welfare
Both firms offer fixed services (Asymmetric Equilibrium)	Consumer surplus = $W_c^* - \dfrac{b\left(16b^4 - 8b\sigma_1\sigma_2^2 + 5\sigma_1^2\sigma_2^2 + 8b^2\sigma_2(\sigma_1 + \sigma_2)\right)}{4\left(4b^2 + 4b\sigma_2 + \sigma_1\sigma_2\right)^2}$ Social welfare = $W_c^* - \dfrac{b\left(5\sigma_1^2\sigma_2^2 + 16b^4 - 16b^3\sigma_2^2 - 4b\sigma_1\sigma_2^2(2 + \sigma_1) + 8b^2\sigma_2\left(\sigma_1 + \sigma_2 - 2\sigma_2^2\right)\right)}{4\left(4b^2 + 4b\sigma_2 + \sigma_1\sigma_2\right)^2}$
Both firms offer fixed services (Symmetric Equilibrium)	Consumer surplus = $W_c^* - \dfrac{b}{12}$ Social welfare = $W_c^* - \dfrac{8b^2 - b(4(\sigma_1 + \sigma_2) - 3)}{36}$
Both firms offer variable services (Symmetric Equilibrium)	Consumer surplus = W_c^* Social welfare = $W_c^* - \dfrac{4b^2 - b(\sigma_1 + \sigma_2)}{8}$
Firm 1 offers variable services & Firm 2 offers fixed services (Asymmetric Equilibrium)	Consumer surplus = $W_c^* - \dfrac{b\left(16b^4 - 8b\sigma_1\sigma_2^2 + 3\sigma_1^2\sigma_2^2 + 8b^2\sigma_2(\sigma_1 + \sigma_2)\right)}{4\left(4b^2 + 4b\sigma_2 + \sigma_1\sigma_2\right)^2}$ Social welfare = $W_c^* - \dfrac{b\left(3\sigma_1^2\sigma_2^2 + 16b^4 - 16b^3\sigma_2^2 - 4b\sigma_1\sigma_2^2(2 + \sigma_1) + 8b^2\sigma_2\left(\sigma_1 + \sigma_2 - 2\sigma_2^2\right)\right)}{4\left(4b^2 + 4b\sigma_2 + \sigma_1\sigma_2\right)^2}$

Table 3: Welfare under different models of firm competition

Proofs

Part 1 - Existence of pure-strategy equilibrium

Let I denote the set of players in a non-cooperative game. For each agent $i \in I$, let the strategy set be defined as $S_i \subset \mathbb{R}$. Notice that S_i is a closed-interval, non-empty vector with no gap, which implies that the strategy space is convex $(s_i = \theta x_i + (1-\theta)y_i \in S_i \ \forall\theta \in [0,1];\ \ x_i, y_i \in S_i)$. S_i is compact because it is both closed $(S_i = \{s_i \in S_i \mid 0 \le s_i \le b\})$ and bounded $(S_i \subset \mathbb{R}$, $\exists r \in \mathbb{R}$ such that $\|s_i\| < r \ \forall s_i \in S_i)$. Further, let S be the Cartesian product $\Pi_{i \in I} S_i$. The payoff function of each agent is $\pi_i : S \to \mathbb{R}$. Using the notation of Baye et al. (1993), the aggregator function $\Phi : S \times S \to \mathbb{R} \cup \{\pm\infty\}$ is given by:

$$\Phi(x,y) = \sum_{i \in I} \pi_i(x_i, y_{-i}) \tag{A.1}$$

where $x, y \in S$ and $x_i \in S_i, y_{-i} \in S_{-i}$. In this proof, we shall show that our aggregator function satisfies the diagonal transfer quasiconcavity (d.t.q.) and diagonal transfer continuity (d.t.c.) conditions that warrant the existence of pure-strategy equilibrium even when the payoff functions are discontinuous (Baye et al. 1993).

Let A_i, B_i and C_i be three non-empty subsets of S_i that denote the different *sets* of strategies that player i may adopt. In particular:

$$
\begin{aligned}
A_i &= \{s_i \in S_i \mid s_i < s_{-i}, \forall s_{-i} \in S_{-i}\} \\
B_i &= \{s_i \in S_i \mid s_i = s_{-i}, \forall s_{-i} \in S_{-i}\} \\
C_i &= \{s_i \in S_i \mid s_i > s_{-i}, \forall s_{-i} \in S_{-i}\}
\end{aligned}
\tag{A.2}
$$

represent the subset of services that belong to regions a, b, and c throughout this paper. Hence the aggregator function can be defined by the corresponding service levels of the two firms:

$$
\begin{aligned}
\Phi(x,y) &: A_i \times C_{-i} \\
\Phi(y,y) &: B_i \times B_{-i}
\end{aligned}
\tag{A.3}
$$

When both firms offer fixed services[3]: To show that $\Phi(x,y) = \pi_{ia} + \pi_{-ic}$ is d.t.q. in x and d.t.c., we shall first demonstrate that $\Phi(x,y)$ is concave in s_i:

$$
\Phi(x,y) = -\frac{bs_{-i}^{\,2} + bs_i^{\,2} - \sigma_i s_i s_{-i} + \sigma_{-i} s_i s_{-i} - b\sigma_{-i} s_{-i} + \sigma_{-i} s_{-i}^{\,2}}{b}
\tag{A.4}
$$

It can be easily verified that the second order condition of $\Phi(x,y)$ w.r.t. s_i is negative. Therefore, by proposition 1 in BTZ, the aggregator function is diagonally transfer quasiconcave in s_i. Second, notice that $\Phi(x,y)$ is continuous. Therefore, by proposition 2 in Baye et al. , the aggregator function is also diagonally transfer continuous. By theorem 1, the game possesses a pure-strategy Nash equilibrium.

Part II: Proof of Lemmas

<u>Proof of Lemma 1 & 2: Equilibria when both firms offer fixed services</u>

The profit function of Firm 1 (and by symmetry, Firm 2) can be expressed as:

$$
\pi_1^F = \begin{cases}
\pi_{1a}^F = \sigma_1 s_1 \left(\dfrac{s_2}{b}\right) - s_1^{\,2} & \text{if } (s_1 < s_2) \\
\pi_{1b}^F = \dfrac{\sigma_1 s_1 (b - s_1)}{2b} - s_1^{\,2} & \text{if } (s_1 = s_2) \\
\pi_{1c}^F = \dfrac{\sigma_1 s_1 (b - s_1 - s_2)}{b} - s_1^{\,2} & \text{if } (s_1 > s_2)
\end{cases}
\tag{A5}
$$

[3] Proof of existence of equilibrium for the remaining cases is available from the authors.

(i) Let there be an asymmetric equilibrium given by $\left\{s_1^{F*}, s_2^{F*}\left(s_1^{F*} \neq s_2^{F*}\right)\right\}$. Recall that to be an equilibrium strategy, s_1^{F*} must maximize $\pi_1^F\left(s_1, s_2^{F*}\right)$ not only in the interval $s_1 < s_2$ but on the whole set of S_1 (and similarly for Firm 2), i.e.

$$\pi_1^F\left(s_1^{F*}, s_2^{F*}\right) \geq \pi_1^F\left(s_1^F, s_2^{F*}\right) \quad ; \quad \pi_2^F\left(s_1^{F*}, s_2^{F*}\right) \geq \pi_2^F\left(s_1^{F*}, s_2^F\right) \tag{A6}$$

Solving simultaneously the best responses of the two firms in their respective profit regions, we have $\frac{\sigma_1}{2b} s_2 = \frac{(b - s_1)\sigma_2}{2(b + \sigma_2)}$, which yields:

$$\left(s_1^{F*}, s_2^{F*}\right) = \left(\frac{b\sigma_1\sigma_2}{4b(b + \sigma_2) + \sigma_1\sigma_2}, \frac{2b^2\sigma_2}{4b(b + \sigma_2) + \sigma_1\sigma_2}\right) \tag{A7}$$

Putting (A7) into the respective profit functions of the two firms:

$$\left(\pi_1^{F*}, \pi_2^{F*}\right) = \left(\frac{b^2\sigma_1^2\sigma_2^2}{[4b(b + \sigma_2) + \sigma_1\sigma_2]^2}, \frac{4b^3\sigma_2^2(b + \sigma_2)}{[4b(b + \sigma_2) + \sigma_1\sigma_2]^2}\right) \tag{A8}$$

To identify conditions for which (A6) is true, we breakdown the necessary conditions as:

$$\pi_{1a}^F\left(s_1^{F*}, s_2^{F*}\right) \geq \pi_{1b}^F\left(s_1^F = s_2^{F*}\right) \tag{A9}$$

and

$$\pi_{1a}^F\left(s_1^{F*}, s_2^{F*}\right) \geq \pi_{1c}^F\left(s_1^F > s_2^{F*}\right) \tag{A10}$$

for Firm 1. Incorporating the equilibrium services in Firm 1's profit functions and simplifying (A9), we have $\frac{2b^3\sigma_2(2b\sigma_2 - 2b\sigma_1 - \sigma_1\sigma_2)}{\left(4b^2 + 4b\sigma_2 + \sigma_1\sigma_2\right)^2} \geq 0$. This implies $\sigma_2 \geq \frac{2b\sigma_1}{2b - \sigma_1}$. To find the competing firm characteristics that satisfy equation (A10), we shall first eliminate those firm types for whom this condition will not be met i.e., we shall identify σ_1, σ_2 for whom $\pi_{1a}^F\left(s_1^{F*}, s_2^{F*}\right) < \pi_{1c}^F\left(s_1^F > s_2^{F*}\right)$ or as simplified below in equation (A11).

$$s_1^F\left[\sigma_1 - \frac{s_1^F(b + \sigma_1)}{b} - \frac{2b\sigma_1\sigma_2}{4b(b + \sigma_2) + \sigma_1\sigma_2}\right] - \frac{b^2{\sigma_1}^2{\sigma_2}^2}{[4b(b + \sigma_2) + \sigma_1\sigma_2]^2} > 0 \tag{A11}$$

We can reduce (A11) to a quadratic expression of the form $\left(s_1^F - x\right)\left(s_1^F - y\right) < 0$, where

$$x = \frac{b\sigma_1\left(4b^2 + 2b\sigma_2 + \sigma_1\sigma_2 - \sqrt{16b^4 + 16b^3\sigma_2 + 8b^2\sigma_1\sigma_2 + {\sigma_1}^2{\sigma_2}^2}\right)}{2(b + \sigma_1)(4b(b + \sigma_2) + \sigma_1\sigma_2)}$$
$$\text{and} \tag{A12}$$
$$y = \frac{b\sigma_1\left(4b^2 + 2b\sigma_2 + \sigma_1\sigma_2 + \sqrt{16b^4 + 16b^3\sigma_2 + 8b^2\sigma_1\sigma_2 + {\sigma_1}^2{\sigma_2}^2}\right)}{2(b + \sigma_1)(4b(b + \sigma_2) + \sigma_1\sigma_2)}$$

Note that since $4b^2 + 2b\sigma_2 + \sigma_1\sigma_2 > \sqrt{16b^4 + 16b^3\sigma_2 + 8b^2\sigma_1\sigma_2 + \sigma_1^2\sigma_2^2}$, both terms are positive. (A11) implies that either $y < s_1^F < x$ <u>or</u> $x < s_1^F < y$. Since $x < y$, we need to only consider $x < s_1^F < y$. Further, since we are considering $s_1^F > s_2^{F*}$, to eliminate the possibility that $\pi_{1a}^F\left(s_1^{F*}, s_2^{F*}\right) < \pi_{1c}^F\left(s_1^F > s_2^{F*}\right)$ we need to find when $y \le s_2^{F*}$. Using (A7) and (A12):

$$s_2^{F*} - y = \frac{b\left(4b^2\sigma_2 - 4b^2\sigma_1 + 2b\sigma_1\sigma_2 - \sigma_1^2\sigma_2 - \sigma_1\sqrt{16b^4 + 16b^3\sigma_2 + 8b^2\sigma_1\sigma_2 + \sigma_1^2\sigma_2^2}\right)}{2(b+\sigma_1)(4b(b+\sigma_2)+\sigma_1\sigma_2)} \ge 0 \quad \text{(A13)}$$

$$\Rightarrow \sigma_2 \ge \frac{8b^2\sigma_1}{4b^2 - \sigma_1^2} \quad \text{(A14)}$$

Hence the necessary conditions for the existence of an asymmetric equilibrium are given by

$$\sigma_1 < 2b, \quad \sigma_2 \ge \frac{8b^2\sigma_1}{4b^2 - \sigma_1^2} \quad \text{(A15)}$$

Similarly for Firm 2, equations (A16) and (A17) need to be satisfied:

$$\pi_{2c}^F\left(s_1^{F*}, s_2^{F*}\right) \ge \pi_{2b}^F\left(s_2^F = s_1^{F*}\right) \quad \text{(A16)}$$

$$\pi_{2c}^F\left(s_1^{F*}, s_2^{F*}\right) \ge \pi_{2a}^F\left(s_2^F < s_1^{F*}\right) \quad \text{(A17)}$$

Equation (A16) is always true as $\dfrac{b^2\sigma_2^2\left(4b^2 - 2b\sigma_1 + \sigma_1^2 + 4b\sigma_2 - 2\sigma_1\sigma_2\right)}{[4b(b+\sigma_2)+\sigma_1\sigma_2]^2} \ge 0$ for $\sigma_1 < 2b$.

Equation (A17) can be written as $s_2^F\left[\dfrac{\sigma_1\sigma_2^2}{4b(b+\sigma_2)+\sigma_1\sigma_2} - s_2^F\right] - \dfrac{4b^3\sigma_j^2(b+\sigma_j)}{\left[4b(b+\sigma_j)+\sigma_i\sigma_j\right]^2} \le 0$ and reduced to $\left(s_2^F - w\right)\left(s_2^F - z\right) \ge 0$, where

$$w = \frac{4b^2\sigma_1\sigma_2^2 + 4b\sigma_1\sigma_2^3 + \sigma_1^2\sigma_2^2 - \sqrt{\sigma_2^2(4b(b+\sigma_2)+\sigma_1\sigma_2)^2\left(\sigma_1^2\sigma_2^2 - 16b^3\sigma_2 - 16b^4\right)}}{2[4b(b+\sigma_2)+\sigma_1\sigma_2]^2}$$
$$\text{and} \quad \text{(A18)}$$
$$z = \frac{4b^2\sigma_1\sigma_2^2 + 4b\sigma_1\sigma_2^3 + \sigma_1^2\sigma_2^2 + \sqrt{\sigma_2^2(4b(b+\sigma_2)+\sigma_1\sigma_2)^2\left(\sigma_1^2\sigma_2^2 - 16b^3\sigma_2 - 16b^4\right)}}{2[4b(b+\sigma_2)+\sigma_1\sigma_2]^2}$$

Note that since $4b^2\sigma_1\sigma_2^2 + 4b\sigma_1\sigma_2^3 + \sigma_1^2\sigma_2^2 > \sqrt{\sigma_2^2(4b(b+\sigma_2)+\sigma_1\sigma_2)^2\left(\sigma_1^2\sigma_2^2 - 16b^3\sigma_2 - 16b^4\right)}$, both terms are positive. (A17) is satisfied if $z \ge s_2^F \le w$ <u>or</u> $z \le s_2^F \ge w$. Since $w < z$ and $s_2^F < s_1^{F*}$, a sufficient condition is $s_1^{F*} \le w$, which is always true for $\sigma_1 < 2b$:

$$\frac{4b\sigma_1\sigma_2\left(\sigma_2{}^2-b\sigma_2-2b^2\right)+\sigma_1{}^2\sigma_2{}^2\left(\sigma_2-2b\right)-\sqrt{\sigma_2{}^2\left(4b\left(b+\sigma_2\right)+\sigma_1\sigma_2\right)^2\left(\sigma_1{}^2\sigma_2{}^2-16b^3\sigma_2-16b^4\right)}}{2\left[4b\left(b+\sigma_2\right)+\sigma_1\sigma_2\right]^2}\geq 0 \quad \text{(A19)}$$

Hence the conditions identified in equation (A15) are indeed necessary and sufficient. Since we derive the firm parameters endogenously we can see that an asymmetric equilibrium exists only when $\sigma_1 < 2b,\ \ \sigma_2 \geq \dfrac{8b^2\sigma_1}{4b^2-\sigma_1{}^2}$

.

(ii) Let there be a symmetric equilibrium given by $s_1^{F*}=s_2^{F*}$ such that

$$\pi_1^F\left(s_1^{F*},s_2^{F*}\right)\geq\pi_1^F\left(s_1^F,s_2^{F*}\right)\ \ ;\ \ \pi_2^F\left(s_1^{F*},s_2^{F*}\right)\geq\pi_2^F\left(s_1^{F*},s_2^F\right) \quad \text{(A20)}$$

For Firm 1 the above implies $\pi_{1b}^F\left(s_1^{F*},s_2^{F*}\right)\geq\pi_{1a}^F\left(s_1^F<s_2^{F*}\right)$ and $\pi_{1b}^F\left(s_1^{F*},s_2^{F*}\right)\geq\pi_{1c}^F\left(s_1^F>s_2^{F*}\right)$. Let us first consider the case when Firm 1 offers some service higher than $s_1^{F*}=s_2^{F*}$, i.e. some $s_1^F=s_2^{F*}+k$ where $k>0$. In order for $\pi_{1b}^F\left(s_1^{F*},s_2^{F*}\right)\geq\pi_{1c}^F\left(s_1^F>s_2^{F*}\right)$, we need

$$\frac{2bk^2+4bks_2^{F*}-2bk\sigma_1+2k^2\sigma_1-b\sigma_1s_2^{F*}+6k\sigma_1s_2^{F*}+3\sigma_1s_2^{F*2}}{2b}\geq 0 \quad \text{(A21)}$$

To identify conditions necessary for (A21) to hold $\forall k>0$, consider the limit: $\lim\limits_{k\to 0}\dfrac{2bk^2+4bks_2^{F*}-2bk\sigma_1+2k^2\sigma_1-b\sigma_1s_2^{F*}+6k\sigma_1s_2^{F*}+3\sigma_1s_2^{F*2}}{2b}=3\sigma_1s_2^{F*2}-b\sigma_1s_2^{F*}$. We can see that $s_2^{F*}\geq\dfrac{b}{3}$ is both necessary and sufficient. Similarly, consider the case where the firm offers some service less than $s_1^{F*}=s_2^{F*}$, i.e. some $s_1^F=s_2^{F*}-l$ where $l>0$. In order for $\pi_{1b}^F\left(s_1^{F*},s_2^{F*}\right)\geq\pi_{1a}^F\left(s_1^F<s_2^{F*}\right)$, we need

$$\frac{2bl^2-4bls_2^{F*}+b\sigma_1s_2^{F*}+2l\sigma_1s_2^{F*}-3\sigma_1s_2^{F*2}}{2b}\geq 0 \quad \text{(A22)}$$

Again, we need to establish that $\pi_{1b}^F\left(s_1^{F*},s_2^{F*}\right)\geq\pi_{1c}^F\left(s_1^F>s_2^{F*}\right)\quad\forall k>0$. Since $\lim\limits_{k\to 0}\dfrac{2bl^2-4bls_2^{F*}+b\sigma_1s_2^{F*}+2l\sigma_1s_2^{F*}-3\sigma_1s_2^{F*2}}{2b}=b\sigma_1s_2^{F*}-3\sigma_1s_2^{F*2}$, $s_2^{F*}\leq\dfrac{b}{3}$ is both necessary and sufficient for equation (A22) to be true, with the necessary condition that $\sigma_1\geq 2b$. By symmetry we have the conditions for Firm 2. This implies that when both firm parameters are given by $\sigma_1,\sigma_2\geq 2b$, the symmetric equilibrium and respective profits are given by

$$\left(s_1^{F*}, s_2^{F*}\right) = \left(\frac{b}{3}, \frac{b}{3}\right) \quad ; \quad \left(\pi_1^{F*}, \pi_2^{F*}\right) = \left(\frac{b(\sigma_1 - b)}{9}, \frac{b(\sigma_2 - b)}{9}\right) \tag{A23}$$

Since we derive the firm parameters endogenously we can see that the symmetric equilibrium exists iff $\sigma_1, \sigma_2 \geq 2b$.

__Proof of Lemma 3: Symmetric equilibrium when both firms offer variable services__

The profit function of Firm 1 (and by symmetry, Firm 2) can be expressed as:

$$\pi_1^V = \begin{cases} \pi_{1a}^V = \dfrac{\sigma_1 {s_1}^2}{2b} - {s_1}^2 \quad \text{if} \quad (s_1 < s_2) \\ \pi_{1b}^V = \dfrac{\sigma_1 s_1}{2} - \dfrac{(\sigma_1 + 2b)\,{s_1}^2}{2b} \quad \text{if} \quad (s_1 = s_2) \\ \pi_{1c}^V = \sigma_1 s_1 - \dfrac{\sigma_1 {s_2}^2}{2b} - \dfrac{(\sigma_1 + b)\,{s_1}^2}{b} \quad \text{if} \quad (s_1 > s_2) \end{cases} \tag{A24}$$

Assume that there exists a symmetric equilibrium given by $s_1^{NF*} = s_2^{NF*}$. If this pair is indeed an equilibrium then equation (A25) needs to be satisfied.

$$\pi_1^V\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_1^V\left(s_1^V, s_2^{V*}\right) \ ; \ \pi_2^V\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_2^V\left(s_1^{V*}, s_2^V\right) \tag{A25}$$

Breaking down Firm 1's analysis into two cases, i.e. $\pi_{1b}^V\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_{1a}^V\left(s_1^V < s_2^{V*}\right)$ and $\pi_{1b}^V\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_{1c}^V\left(s_1^V > s_2^{V*}\right)$, we first examine the case when Firm 1 offers higher services, i.e. $s_1^V = s_2^{V*} + k$ for some $k > 0$. For $\pi_{1b}^V\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_{1c}^V\left(s_1^V > s_2^{V*}\right)$ we need

$$\frac{2bk^2 + 4bks - 2bk\sigma_1 + 2k^2\sigma_1 - b\sigma_1 s_2^{V*} + 4k\sigma_1 s_2^{V*} + 2\sigma_1 s_2^{V*2}}{2b} \geq 0 \tag{A26}$$

Since we need to establish that $\pi_{1b}^V\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_{1c}^V\left(s_1^V > s_2^{V*}\right) \quad \forall k > 0$, and that

$\lim\limits_{k \to 0} \dfrac{2bk^2 + 4bks - 2bk\sigma_1 + 2k^2\sigma_1 - b\sigma_1 s_2^{V*} + 4k\sigma_1 s_2^{V*} + 2\sigma_1 s_2^{V*2}}{2b} = 2\sigma_1 s_2^{V*2} - b\sigma_1 s_2^{V*} \quad , \quad s_2^{V*} \geq \dfrac{b}{2}$ is both necessary and sufficient for equation (A26) to be true. Similarly if the firm considers services less than $s_1^{V*} = s_2^{V*}$, i.e. $s_1^{NF} = s_2^{NF*} - l$ for some $l > 0$, then for $\pi_{1b}^V\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_{1a}^V\left(s_1^V < s_2^{V*}\right)$, we need

$$\frac{2bl^2 - 4bls_2^{V*} - l^2\sigma_1 + b\sigma_1 s_2^{V*} + 2l\sigma_1 s_2^{V*} - 2\sigma_1 s_2^{V*2}}{2b} \geq 0 \tag{A27}$$

Again, we need to establish that $\pi_{1b}^{V}\left(s_1^{V*}, s_2^{V*}\right) \geq \pi_{1c}^{V}\left(s_1^{V} > s_2^{V*}\right) \quad \forall k > 0$. Since $\lim_{k \to 0} \frac{2bl^2 - 4bls_2^{V*} - l^2\sigma_1 + b\sigma_1 s_2^{V*} + 2l\sigma_1 s_2^{V*} - 2\sigma_1 s_2^{V*2}}{2b} = b\sigma_1 s_2^{V*} - 2\sigma_1 s_2^{V*2}$, $s_2^{V*} \geq \frac{b}{2}$ is both necessary and sufficient for equation (A27) to be true, with the necessary condition that $\sigma_1 > 2b$. Notice that $\frac{b}{2}$ is the maximum level of service that any firm would offer in the market as the s_c^* of the consumer with the highest $\frac{p}{r}$ ratio is $\frac{b}{2}$, i.e., there is no consumer in the market will use any services greater than this bound. We can derive Firm 2's strategy by symmetry and hence conclude that a symmetric equilibrium exists for firm parameters given by $\sigma_1, \sigma_2 > 2b$, where the equilibrium service-pair and the respective profits of the two firms are

$$\left(s_1^{V*}, s_2^{V*}\right) = \left(\frac{b}{2}, \frac{b}{2}\right) \quad ; \quad \left(\pi_1^{V*}, \pi_2^{V*}\right) = \left(\frac{b(\sigma_1 - 2b)}{8}, \frac{b(\sigma_2 - 2b)}{8}\right) \tag{A28}$$

Notice that when $\sigma_1 < 2b$, the firm will never opt to offer services less than his competitor as π_{1a}^{V} is negative. By symmetry this implies that when $\sigma_2 < 2b$, Firm 2 will also not consider offering services less than his competitor and thus there can be no asymmetric equilibrium. Further, from our earlier discussion we know that for symmetric equilibrium, the non-negative profit condition requires that a firm's marginal value for information be greater than or equal to $2b$ and hence equilibrium fail to exist when $\sigma_1, \sigma_2 < 2b$.

<u>**Proof of Lemma 4: Asymmetric equilibrium when only one firm offers variable services**</u>

The profit functions of Firm 1 and Firm 2) can be respectively expressed as:

$$\pi_1^{\widetilde{V}} = \begin{cases} \pi_{1a}^{\widetilde{V}} = s_1\left(\frac{\sigma_1 s_2}{b} - s_1\right) & \text{if } (s_1 < s_2) \\ \pi_{1b}^{\widetilde{V}} = \frac{s_1(\sigma_1 - 2s_1)}{2} & \text{if } (s_1 = s_2) \\ \pi_{1c}^{\widetilde{V}} = \sigma_1 s_1 - \frac{(\sigma_1 + b)s_1^2}{b} & \text{if } (s_1 > s_2) \end{cases} \tag{A29}$$

$$\pi_2^{\widetilde{F}} = \begin{cases} \pi_{2a}^{\widetilde{F}} = \frac{\sigma_2 s_2 (b - s_1 - s_2)}{b} - s_2^2 & \text{if } (s_1 < s_2) \\ \pi_{2b}^{\widetilde{F}} = \frac{\sigma_2 s_2}{2} - \frac{(\sigma_2 + b)s_2^2}{b} & \text{if } (s_1 = s_2) \\ \pi_{2c}^{\widetilde{F}} = -s_2^2 & \text{if } (s_1 > s_2) \end{cases} \tag{A30}$$

Recall that there can be no symmetric equilibrium as the variable-service firm can always corner the entire market by offering a service that is equal to or higher than that offered by the fixed-service competitor. And since there is no equilibrium in the region $s_1 > s_2$, if there is any asymmetric equilibrium it can only exist in $s_1 < s_2$.

Let $\left\{s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\left(s_1^{\widetilde{V}*} < s_2^{\widetilde{F}*}\right)\right\}$ be an equilibrium pair. To be an equilibrium strategy, $s_1^{\widetilde{V}*}$ must maximize $\pi_1^{\widetilde{V}}\left(s_1^{\widetilde{V}}, s_2^{\widetilde{F*}}\right)$ not only on the interval $s_1^{\widetilde{V}} < s_2^{\widetilde{F*}}$ but on the whole domain of $s_1^{\widetilde{V}}$, i.e.,

$$\pi_1^{\widetilde{V}}\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right) \geq \pi_1^{\widetilde{V}}\left(s_1^{\widetilde{V}}, s_2^{\widetilde{F}*}\right) \quad ; \quad \pi_2^{\widetilde{F}}\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right) \geq \pi_2^{\widetilde{F}}\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}}\right) \tag{A31}$$

Solving simultaneously the best responses of the two firms in their respective profit regions, we have $\frac{\sigma_1}{2b} s_2 = \frac{(b - s_1)\sigma_2}{2(b+\sigma_2)}$, which yields:

$$\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right) = \left(\frac{b\sigma_1\sigma_2}{4b(b+\sigma_2) + \sigma_1\sigma_2}, \frac{2b^2\sigma_2}{4b(b+\sigma_2) + \sigma_1\sigma_2}\right) \tag{A32}$$

Putting (A32) into the respective profit functions of the two firms:

$$\left(\pi_1^{\widetilde{V}*}, \pi_2^{\widetilde{F}*}\right) = \left(\frac{b^2\sigma_1^2\sigma_2^2}{[4b(b+\sigma_2) + \sigma_1\sigma_2]^2}, \frac{4b^3\sigma_2^2(b+\sigma_2)}{[4b(b+\sigma_2) + \sigma_1\sigma_2]^2}\right) \tag{A33}$$

Now we need to verify that equation (A31) is indeed true. For Firm 1, we can breakdown the necessary condition as

$$\pi_{1a}^{\widetilde{V}}\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right) \geq \pi_{1b}^{\widetilde{V}}\left(s_1^{\widetilde{V}} = s_2^{\widetilde{F}*}\right) \tag{A34}$$

and

$$\pi_{1a}^{\widetilde{NF}}\left(s_1^{\widetilde{NF}*}, s_2^{\widetilde{F}*}\right) \geq \pi_{1c}^{\widetilde{NF}}\left(s_1^{\widetilde{NF}} > s_2^{\widetilde{F}*}\right) \tag{A35}$$

Incorporating the equilibrium services in Firm 1's profit functions given in (A33) and simplifying (A34), we have $\frac{4b^3\sigma_2(b\sigma_2 - b\sigma_1 - \sigma_1\sigma_2)}{\left(4b^2 + 4b\sigma_2 + \sigma_1\sigma_2\right)^2} \geq 0$. And since $\sigma_2 > 0$, this implies

$$\sigma_1 < b, \sigma_2 \geq \frac{b\sigma_1}{b - \sigma_1} \tag{A36}$$

In order to identify conditions for (A35) to hold, let Firm 1 offer $s_2^{\widetilde{F}*} + k$ for some $k > 0$. We need to find firm parameters such that $\pi_{1a}^{\widetilde{V}}\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right) \geq \pi_{1c}^{\widetilde{V}}\left(s_1^{\widetilde{V}} > s_2^{\widetilde{F}*}\right)$, i.e.,

$$\begin{aligned}&\frac{b^2\sigma_1^2\sigma_2^2}{[4b(b+\sigma_2) + \sigma_1\sigma_2]^2} - \left\{-\frac{1}{b}\left[\left(k + \frac{2b^2\sigma_2}{4b(b+\sigma_2) + \sigma_1\sigma_2}\right)\right.\right. \\ &\left.\left.*\left(\sigma_1 \cdot \left(k + \frac{2b^2\sigma_2}{4b(b+\sigma_2) + \sigma_1\sigma_2}\right) + b \cdot \left(k - \sigma_1 + \frac{2b^2\sigma_2}{4b(b+\sigma_2) + \sigma_1\sigma_2}\right)\right)\right]\right\} \geq 0\end{aligned} \tag{A37}$$

Simplifying equation (A37) and ensuring non-negative k, we get

$$\sigma_1 < \frac{2b}{1+\sqrt{2}}, \sigma_2 \geq \frac{8b^2\sigma_1}{4b^2 - 4b\sigma_1 - {\sigma_1}^2} \tag{A38}$$

We now need to ensure that the equilibrium is valid from the second firm's perspective by verifying the second part of (A31). As one region is infeasible, we need to only verify that $\pi_{2c}^{\widetilde{F}}\left(s_1^{\widetilde{V}*}, s_2^{\widetilde{F}*}\right) \geq \pi_{2b}^{\widetilde{F}}\left(s_2^{\widetilde{F}} = s_1^{\widetilde{V}*}\right)$ which is always true for the equilibrium set of services. Therefore, the necessary and sufficient condition for the asymmetric equilibrium is given by (A38), which also satisfies (A36).

Proof of Lemma 5: Welfare analysis

In the case of fixed services, both firms offer $s_1^{F*} = s_2^{F*} = \frac{b}{3}$. For notation simplicity we denote this service level as s^{F*}. Consumers with break-even level of services less than the equilibrium service level $\left(s_c^0 \leq \frac{b}{3}\right)$, would not use any services. The disutilities of these consumers are given by the summation of their respective s_c^*. The remaining consumers suffer disutilities depending on the relative distances of their respective s_c^* and s^{F*}. Therefore, the total disutilities suffered by consumers are given by:

$$\int_0^{\frac{s^{F*}}{2}} x \cdot U(s_c^*)dx + \int_{\frac{s^{F*}}{2}}^{s^{F*}} (s^{F*} - x)U(s_c^*)dx + \int_{s^{F*}}^{\frac{b}{2}} (x - s^{F*})U(s_c^*)dx = \frac{b}{12} \tag{A.39}$$

Consumer welfare is given by:

$$w_c = w_c^* - \frac{b}{12} \tag{A.40}$$

where w_c^* denotes the maximum attainable consumer welfare. Since firm surplus is given by $\pi_1^{F*} + \pi_2^{F*}$, the total welfare under fixed services is:

$$W^F = w_c^* - \frac{b}{12} + \frac{b(\sigma_1 - b)}{9} + \frac{b(\sigma_2 - b)}{9} \tag{A.41}$$

$$= \frac{-8b^2 + 36W_c^* + b(4(\sigma_1 + \sigma_2) - 3)}{36} \tag{A.42}$$

In the case of variable services, both firms offer $s_1^{V*} = s_2^{V*} = \frac{b}{2}$. The service level offered by the firms satisfies even the consumer with highest demand for personalization. Since consumers are free to choose the level of personalization to adopt and all consumers enjoy their respective optimal level of services, all consumers attain their highest utilities, i.e. $w_c = w_c^*$.

Since firm surplus is $\pi_1^{V*} + \pi_2^{V*}$, and the total welfare under variable services is:

$$W^V = w_c^* + \frac{b(\sigma_1 - 2b)}{8} + \frac{b(\sigma_2 - 2b)}{8} \tag{A.43}$$

$$= \frac{-4b^2 + 8W_c^* + b(\sigma_1 + \sigma_2)}{8} \tag{A.44}$$

By comparing (A.42) and (A.44), social welfare is higher under variable services if $\sigma_1 + \sigma_2 > 20b - 6$. Further, from (A23) and (A28), we know that $\pi_i^{V*} > \pi_i^{F*}$ if $\sigma_i > 10b$, $i = \{1,2\}$.